The Capital Asset Pricing Model

Capital Budgeting and Corporate Objectives

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Overview

- Utility and risk aversion
  - Choosing efficient portfolios
- Investing with a risk-free asset
  - Borrowing and lending
  - The market portfolio
  - The Capital Market Line (CML)
- The Capital Asset Pricing Model (CAPM)
  - The Security Market Line (SML)
  - Beta
  - Project analysis
Introducing a Riskfree Asset

- Suppose we introduce the opportunity to invest in a riskfree asset.
  » How does this alter investors’ portfolio choices?

- The riskfree asset has a zero variance, and zero covariance with every other asset (or portfolio).
  » \( \text{var}(r_f) = 0. \)
  » \( \text{cov}(r_p, r_j) = 0 \) for all \( j \).

- What is the expected return and variance of a portfolio consisting of a fraction \((1-\alpha)\) of the riskfree asset and a fraction \(\alpha\) of the risky asset (or portfolio of risky assets)?
Risk and Return with a Riskfree asset

- Expected Return
  \[ E[r_p] = \alpha E[r_j] + (1 - \alpha) r_f \]

- Variance and Standard Deviation
  \[ Var[r_p] = \sigma_p^2 = \alpha^2 \sigma_j^2 \Rightarrow \sigma_p = \alpha \sigma_j \]

- Hence, the risk-return tradeoff is:
  \[ E[r_p] = r_f + \frac{(E[r_j] - r_f)}{\sigma_j} \sigma_p \]
  » The slope is called the Sharpe-Ratio

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Risk and Return with a Riskfree asset

- The line represents all portfolios depending on \( \alpha \)
Investing with Borrowing and Lending

Expected Return

\( E[r_M] \)

\( r_f \)

\( \alpha = 0 \)

\( \alpha = 0.5 \)

\( \alpha = 1 \)

\( \alpha = 2 \)

Lending

Borrowing

Standard Deviation

\( \sigma_M \)

The Capital Market Line I

Expected Return

\( r_f \)

\( M \)

IBM

Dow Utility Index

Standard Deviation
The Capital Market Line

The CML gives the tradeoff between risk and return for portfolios consisting of the riskfree asset and the tangency portfolio M. The equation of the CML is:

\[ E(r_p) = r_f + \sigma_p \frac{E(r_M) - r_f}{\sigma_M} \]

The expected rate of return on a risky asset can be thought of as composed of two terms.

» The return on a riskfree security, like U.S. Treasury bills; compensating investors for the time value of money.

» A risk premium to compensate investors for bearing risk.

\[ E(r_p) = r_f + \text{Risk x Market Price of Risk} \]
Decomposition of Return

What is the “M” Portfolio?

- Everybody holds the tangency portfolio M
  - If all hold the same portfolio, it must be the market!
- Nobody can do better than holding the market
- Write the weight of asset j in the market portfolio as $w_j$.
  - Then we have:

$$E(r_M) = \sum_{j=1}^{J} w_j (E(r_j) - r_f) + r_f$$

$$Var(r_M) = \sum_{i=1}^{I} \sum_{j=1}^{J} w_i w_j Cov(r_j, r_i)$$
All Risk-Return Tradeoffs are Equal

- The excess return on an asset is:
  \[ (E(r_j) - r_f) \]
- Recall the relevant measure of asset risk relative to a well-diversified portfolio is the covariance. Thus, the risk of the asset is
  \[ \text{Cov} (r_j, r_M) \]
- Thus, the return per unit of risk is simply the ratio:
  \[ \frac{E(r_j) - r_f}{\text{Cov} (r_j, r_M)} \]
- This measure must be the same for all assets!
  » Consider what happens when this is not the case
  \[
  \frac{E(r_A) - r_f}{\text{Cov}(r_A, r_M)} > \frac{E(r_B) - r_f}{\text{Cov}(r_B, r_M)}
  \]

The Capital Asset Pricing Model

- If the risk-return tradeoff is the same for all assets, then it is the one of the market:
  \[
  \frac{E(r_A) - r_f}{\text{Cov}(r_A, r_M)} = \frac{E(r_B) - r_f}{\text{Cov}(r_B, r_M)} = \frac{E(r_M) - r_f}{\text{Var}(r_M)}
  \]
  This gives the relationship between risk and expected return for individual stocks and portfolios.
  » This is called the **Security Market Line**.
  \[
  E(r_A) = r_f + \frac{\text{Cov}(r_A, r_M)}{\text{Var}(r_M)}(E(r_M) - r_f) = r_f + \beta_A(E(r_M) - r_f)
  \]
  where
  \[
  \beta_A = \frac{\text{Cov}(r_A, r_M)}{\text{Var}(r_M)}
  \]
Capital Asset Pricing Model
A Graphical Illustration

\[
E(r) = r_f + \beta \left( E(r_M) - r_f \right)
\]

The CML and SML

\[
E(r_i) = r_f + \beta_i \left( E(r_M) - r_f \right)
\]
The SML and mispriced stocks

- Suppose for a particular stock:
  
  $$E(r_j) < r_f + \frac{\text{Cov}(r_j, r_M)}{\text{Var}(r_M)}[E(r_M) - r_f]$$

- Remember the definition of expected returns:
  
  $$E(r_j) = \frac{E(P_j^1 + D_j^1) - P_j^0}{P_j^0}$$

- Then $P_0$ falls, so that $E(r_j)$ increases until disequilibrium vanishes and the equation holds!

The SML and mispriced stocks

- Stock $j$ is overvalued at $X$:
  - price drops,
  - expected return rises.

- At $Y$, stock $j$ would be undervalued!
  - expected return falls
  - price increases
The Capital Asset Pricing Model

- The appropriate measure of risk for an individual stock is its beta.
- Beta measures the stock’s sensitivity to market risk factors.
  » The higher the beta, the more sensitive the stock is to market movements.
- The average stock has a beta of 1.0.
- Portfolio betas are weighted averages of the betas for the individual stocks in the portfolio.

Beta and Standard Deviation

Risk of a Share (Variance) = Market risk of the share + Specific risk of the share

Beta of share x Risk of market

Risk of a portfolio = Market risk of the portfolio + Specific risk of the portfolio

Beta of Portfolio x Risk of market

This is the major element of a share's risk
This is negligible for a diversified portfolio
Using Regression Analysis to Measure Betas

Betas of Selected Common Stocks
(based on monthly returns)

<table>
<thead>
<tr>
<th>Stock</th>
<th>Beta (5 years)</th>
<th>Beta (10 years)</th>
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</thead>
<tbody>
<tr>
<td>GENERAL MOTORS CORP</td>
<td>1.18</td>
<td>1.10</td>
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<tr>
<td>BOEING CO</td>
<td>0.98</td>
<td>0.70</td>
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<tr>
<td>BRISTOL MYERS SQUIBB CO</td>
<td>0.59</td>
<td>0.56</td>
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<tr>
<td>DELTA AIR LINES INC</td>
<td>2.54</td>
<td>1.51</td>
</tr>
<tr>
<td>CONTINENTAL AIRLINES INC</td>
<td>3.60</td>
<td>2.15</td>
</tr>
<tr>
<td>HEWLETT PACKARD CO</td>
<td>1.72</td>
<td>1.70</td>
</tr>
<tr>
<td>DOW CHEMICAL CO</td>
<td>0.94</td>
<td>0.73</td>
</tr>
<tr>
<td>EXXON CORP</td>
<td>0.57</td>
<td>0.44</td>
</tr>
<tr>
<td>MERCK &amp; CO INC</td>
<td>0.40</td>
<td>0.37</td>
</tr>
<tr>
<td>A T &amp; T CORP</td>
<td>0.84</td>
<td>0.95</td>
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<tr>
<td>HOME DEPOT INC</td>
<td>1.38</td>
<td>1.18</td>
</tr>
<tr>
<td>MCDONALDS CORP</td>
<td>1.01</td>
<td>0.73</td>
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<tr>
<td>MICROSOFT CORP</td>
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<tr>
<td>APPLE COMPUTER INC</td>
<td>1.50</td>
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</tr>
<tr>
<td>GOOGLE INC</td>
<td>-0.07</td>
<td>-0.07</td>
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<tr>
<td>WAL MART STORES INC</td>
<td>0.58</td>
<td>0.67</td>
</tr>
<tr>
<td>JPMORGAN CHASE &amp; CO</td>
<td>1.63</td>
<td>1.52</td>
</tr>
<tr>
<td>YAHOO INC</td>
<td>2.80</td>
<td>3.22</td>
</tr>
</tbody>
</table>
# Beta and Volatility

*(based on monthly returns)*

<table>
<thead>
<tr>
<th>Company</th>
<th>Beta</th>
<th>Total StDev</th>
<th>Specific Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>GENERAL MOTORS CORP</td>
<td>1.18</td>
<td>10.01</td>
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<tr>
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<td>9.03</td>
<td>7.9</td>
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<td>7.87</td>
<td>7.41</td>
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<td>21.06</td>
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<td>11.63</td>
<td>8.72</td>
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<td>DOW CHEMICAL CO</td>
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<td>7.81</td>
<td>6.59</td>
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<tr>
<td>EXXON CORP</td>
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<td>5.45</td>
<td>4.83</td>
</tr>
<tr>
<td>MERCK &amp; CO INC</td>
<td>0.4</td>
<td>8.05</td>
<td>7.85</td>
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<tr>
<td>A T &amp; T CORP</td>
<td>0.84</td>
<td>10.34</td>
<td>9.62</td>
</tr>
<tr>
<td>HOME DEPOT INC</td>
<td>1.38</td>
<td>8.69</td>
<td>6.1</td>
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<td>MCDONALDS CORP</td>
<td>1.01</td>
<td>7.92</td>
<td>6.52</td>
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<td>MICROSOFT CORP</td>
<td>1.18</td>
<td>9.16</td>
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<tr>
<td>APPLE COMPUTER INC</td>
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<td>13.69</td>
<td>11.95</td>
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<td>GOOGLE INC</td>
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<td>14.47</td>
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<td>WAL MART STORES INC</td>
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<td>9.52</td>
<td>6.16</td>
</tr>
<tr>
<td>YAHOO INC</td>
<td>2.8</td>
<td>17.39</td>
<td>12.67</td>
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</tbody>
</table>

Market StDev over this 5 year period was 4.47%

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# Estimating the Expected Rate of Return on Equity

- The SML gives us a way to estimate the expected (or required) rate of return on equity.

\[
E(r_j) = r_f + \beta_j \left[ E(r_M) - r_f \right]
\]

- We need estimates of three things:
  - Riskfree interest rate, \( r_f \):
    - The **riskfree rate** can be estimated by the current yield on one-year Treasury bills.
  - Excess return of market, \([E(r_M)-r_f]\):
    - The **market excess return** can be estimated by looking at the historical difference between the return on stocks and the return on Treasury bills.
  - Beta for the stock, \(\beta_j\):
    - The **betas** are estimated by regression analysis. They are also published in beta books (e.g., Value Line, Merrill Lynch).
Estimating the Expected Rate of Return on Equity

\[ E(r) = 5.2\% + \beta \times (8.6\%) \]

<table>
<thead>
<tr>
<th>Company</th>
<th>Beta</th>
<th>E(r)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GENERAL MOTORS CORP</td>
<td>1.18</td>
<td>15.35</td>
</tr>
<tr>
<td>BOEING CO</td>
<td>0.98</td>
<td>13.63</td>
</tr>
<tr>
<td>BRISTOL MYERS SQUIBB CO</td>
<td>0.59</td>
<td>10.27</td>
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<tr>
<td>DELTA AIR LINES INC</td>
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<td>27.04</td>
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<tr>
<td>CONTINENTAL AIRLINES INC</td>
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<td>HEWLETT PACKARD CO</td>
<td>1.72</td>
<td>19.99</td>
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<tr>
<td>DOW CHEMICAL CO</td>
<td>0.94</td>
<td>13.28</td>
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<tr>
<td>EXXON CORP</td>
<td>0.57</td>
<td>10.10</td>
</tr>
<tr>
<td>MERCK &amp; CO INC</td>
<td>0.4</td>
<td>8.64</td>
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<tr>
<td>A T &amp; T CORP</td>
<td>0.84</td>
<td>12.42</td>
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<td>HOME DEPOT INC</td>
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<td>17.07</td>
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<tr>
<td>MCDONALDS CORP</td>
<td>1.01</td>
<td>13.89</td>
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<td>MICROSOFT CORP</td>
<td>1.18</td>
<td>15.35</td>
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<td>APPLE COMPUTER INC</td>
<td>1.5</td>
<td>18.10</td>
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<tr>
<td>GOOGLE INC</td>
<td>-0.07</td>
<td>4.60</td>
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<td>WAL MART STORES INC</td>
<td>0.58</td>
<td>10.19</td>
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<tr>
<td>JPMORGAN CHASE &amp; CO</td>
<td>1.63</td>
<td>19.22</td>
</tr>
<tr>
<td>YAHOO INC</td>
<td>2.8</td>
<td>29.28</td>
</tr>
</tbody>
</table>

Example of Portfolio Betas and Expected Returns

- Maintaining the assumption that the risk free rate is 5.2% and the risk premium is 8.6% : what is the beta and expected rate of return of an equally-weighted portfolio consisting of Exxon (\( \beta = 0.57 \)) and AT&T (\( \beta = 0.84 \))?

- How would you construct a portfolio with the same beta and expected return, but with the lowest possible standard deviation?

- Use the figure on the following page to locate the minimum variance portfolio with the same expected return.
Example: Mispriced Portfolio

- You have information that the equal weighted portfolio of Exon and AT&T has an expected return that is lower than 11.3%.
  » How should you take advantage of this?
  » Is this an arbitrage opportunity?
  » What risk can you hedge here?
Example: Mispriced Portfolio

- **Buy cheap:**
  \[ w_m = 0.71 \]
  \[ (1 - w_m) = \frac{0.29}{1} \]
  \[ \beta = 0.71 \]

- **Sell expensive:**
  \[ w_{Exxon} = -0.5 \]
  \[ w_{AT&T} = -0.5 \]
  \[ \beta = -0.71 \]

- **Overall Investment:**
  \[ 1 - 1 = 0 \]

- **Overall \( \beta \):**
  \[ 0.71 - 0.71 = 0 \]

Example

- The S&P500 Index has a standard deviation of about 12% per year.
- Gold mining stocks have a standard deviation of about 24% per year and a correlation with the S&P500 of about \( \rho = 0.15 \).
- If the yield on U.S. Treasury bills is 5.2% and the market risk premium is \( [E(r_M) - r_f] = 8.6\% \), what is the expected rate of return on gold mining stocks?
Example

- The beta for gold mining stocks is calculated as follows:

- The expected rate of return on gold mining stocks is:

- What portfolio has the same expected return as gold mining stocks, but the lowest possible standard deviation?

Graphical Illustration

[Diagram with labeled axes and points: E(r) on the y-axis, M and Gold M on the graph, SML and CML lines, and percentage values like 5.2%, 7.8%, 13.8%.]
Using the CAPM for Project Evaluation

- Suppose Microsoft is considering an expansion of its current operations.
  - The expansion will cost $140 million today
  - Expected to generate a net cash flow of $25 million per year for the next 20 years.
  - What is the appropriate risk-adjusted discount rate for the expansion project?
  - What is the NPV of Microsoft’s investment project?
  - Disregard the effect of leverage

Microsoft’s Expansion Project

- The risk-adjusted discount rate for the project, $r_p$, can be estimated by using Microsoft’s beta and the CAPM.

$$ r_p = r_f + \beta (E[r_m] - r_f) = 0.052 + 1.18(0.086) = 0.1535 $$

- Thus, the NPV of the project is:

$$ NPV = \frac{25}{0.1535} \left( 1 - \frac{1}{1.1535^{20}} \right) - 140 = 13.502M $$
Company Risk Versus Project Risk

- The company-wide discount rate is the appropriate discount rate for evaluating investment projects that have the same risk as the firm as a whole.

- For investment projects that have different risk from the firm’s existing assets, the company-wide discount rate is not the appropriate discount rate.

- In these cases, we must rely on industry betas for estimates of project risk.

Company Risk versus Project Risk

- Suppose Microsoft is considering investing in the development of a new airline.
  - What is the risk of this investment?

  - What is the appropriate risk-adjusted discount rate for evaluating the project?

  - Suppose the project offers a 17% rate of return. Is the investment a good one for Microsoft?
Industry Asset Betas

<table>
<thead>
<tr>
<th>Industry</th>
<th>Beta</th>
<th>Industry</th>
<th>Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Airlines</td>
<td>1.80</td>
<td>Agriculture</td>
<td>1.00</td>
</tr>
<tr>
<td>Electronics</td>
<td>1.60</td>
<td>Food</td>
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<tr>
<td>Consumer Durables</td>
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<td>Liquor</td>
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<td>Producer Goods</td>
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<td>Banks</td>
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<tr>
<td>Chemicals</td>
<td>1.25</td>
<td>International Oils</td>
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<tr>
<td>Shipping</td>
<td>1.20</td>
<td>Tobacco</td>
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<tr>
<td>Steel</td>
<td>1.05</td>
<td>Telephone Utilities</td>
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<td>Containers</td>
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<td>Energy Utilities</td>
<td>0.60</td>
</tr>
<tr>
<td>Nonferrous Metals</td>
<td>1.00</td>
<td>Gold</td>
<td>0.35</td>
</tr>
</tbody>
</table>


Company Risk versus Project Risk

- The project risk is closer to the risk of other airlines than it is to the risk of Microsoft’s software business.
- The appropriate risk-adjusted discount rate for the project depends upon the risk of the project. If the average asset beta for airlines is 1.8, then the project’s cost of capital is:

  \[
  r_p = r_f + \beta_p \left( E[r_m] - r_f \right)
  \]

  \[ r_p = \]
**Company Risk versus Project Risk**

![Graph showing the relationship between Company Beta, Project Beta, and Required Return](graph.png)

### Project Evaluation: Rules

- The risk of an investment project is given by the *project’s beta*.
  - Can be different from company’s beta
  - Can often use industry as approximation

- The **Security Market Line** provides an estimate of an appropriate discount rate for the project based upon the project’s beta.
  - Same company may use different discount rates for different projects

- This discount rate is used when computing the project’s net present value.
Summary

- Optimal investments depend on trading off risk and return
  » Investors with higher risk tolerance invest more in risky assets
  » Only risk that can’t be diversified counts
- If investors can borrow and lend, then everybody holds a combination of two portfolios
  » The market portfolio of all risky assets
  » The riskless asset
    – Covariance with the market portfolio counts
- In equilibrium, all stocks must lie on the security market line
  » Beta measures the amount of non-diversifiable risk
  » Expected returns reflect only market risk
  » Use these as required returns in project evaluation