Solutions to Practice Questions

(CAPM)

1. These practice questions are a supplement to the problem sets, and are intended for those of you who want more practice. They are Optional, and are not part of the required material.

2. It is recommended that you look at these problems only after you fully understand how to solve the problem sets, the examples we covered in class, and the ones in the lecture notes.

3. Please note that I have collected these examples from previous teaching material I have had. As such, while in most cases the notation will match the one used in class, the match is not 100%.

4. Some of these questions are easier than the ones you are expected to know how to solve, while others are above the level of knowledge you are expected to show on quizzes and the final.

ENJOY!
1. The expected return for the Alpha firm is given by the Security Market Line: \( r_i = r_f + [r_M - r_f] \beta_i \). We have been given \( \beta_i, r_f, \) and the market risk premium (i.e. \( [r_M - r_f] \)). Plugging into the formula we get: \( r_i = .06 + .085(1.2) = 16.2\% \).

2. The expected return for the Textile Industries is given by the Security Market Line: \( r_i = r_f + [r_M - r_f] \beta_i \). We have been given \( r_f, r_i, \) and the market risk premium (i.e. \( [r_M - r_f] \)). Plugging into the formula we get: \( .142 = .037 + .075 \beta_i \implies \beta_i = 1.4 \).

3. First we can calculate

\[
\beta_i = \frac{\text{Cov}(r_i, r_M)}{\sigma_M^2} = \frac{0.0635}{0.04326} = 1.468
\]

The required return for Durham Co. is then given by the Security Market Line:

\[
 r_i = r_f + [r_M - r_f] \beta_i \\
 = .049 + .094(1.468) \\
= 18.70\%
\]

4. (a) First, we use the CAPM to determine the appropriate discount rate:

\[
r_i = r_f + \beta_i [r_M - r_f] \\
= .04 + .075(0.08) \\
= 10.0\%
\]

The return on the stock is given by \( \frac{\text{DIV} + \text{capital gain}}{\text{price}} = \frac{2 + (P_1 - 50)}{50} \). Setting this equal to .10 and solving for \( P_1 \), we get \( P_1 = $53 \).

(b) If investors believe that the year-end stock price will be $54, then the expected return on the stock is \( \frac{2 + (54 - 50)}{50} = 12\% \) which is greater than the opportunity cost of capital. Another way to see this is to note that the "fair" price today of this stock (the present value of expected cash flows) is \( \frac{2 + 54}{1.10} = $50.91 \), which is greater than the current price. Investors will want to buy the stock, bidding up the price until it reaches $50.91. At that point, the expected return is a "fair" 10\%: \( \frac{2 + (54 - 50.91)}{50.91} = 10\% \).

5. (a) \( r_m - r_f = 0.12 - 0.04 = 0.08 \).

(b) Using the security market line we know that

\[
r = r_f + \beta (r_m - r_f)
\]

Thus, \( r = 0.04 + 1.5(0.12 - 0.04) = 0.16 \).
(c) We use the security market line relationship \( r = r_f + \beta (r_m - r_f) \) to derive an equation for beta:

\[
0.112 = 0.04 + \beta (0.12 - 0.04);
\]

which implies that \( \beta = \frac{0.112 - 0.04}{0.08} = 0.9 \).

6. For the corporate bond the return and the standard deviation are \( r_b = 0.09 \) and \( \sigma_b = 0.1 \) respectively. Similarly, for the S&P500 they are \( r_{SP} = 0.14 \) and \( \sigma_{SP} = 0.16 \) respectively.

(a) All portfolios that are constructed as a combination of the riskless Treasury bill and the index fund that closely tracks the S&P500 satisfy the relation

\[
r_p = r_f + \frac{r_{SP} - r_f}{\sigma_{SP}} \sigma_p,
\]

where \( r_f = 0.06 \) is the rate of return on the riskless Treasury Bill. Thus, for a standard deviation of \( \sigma_p = \sigma_b = 0.1 \) the return one can obtain using a combination of the Treasury Bill and the fund is

\[
r_p = 0.06 + \frac{0.14 - 0.06}{0.16} - 0.1 = 0.06 + \frac{0.08}{0.16} = 0.11.
\]

This is of course better than the 9% you could obtain if you held the corporate bond portfolio.

(b) The return of an investment that has equal amounts in the corporate bond portfolio and the index fund is

\[
r_p = 0.5 r_b + 0.5 r_{SP} = 0.5(0.09) + 0.5(0.14) = 0.115.
\]

On the other hand, letting \( \rho_{b,SP} = 0.1 \) be the correlation between the return on the bond fund and on the index fund, the standard deviation of such a portfolio would be

\[
\sigma_p = \sqrt{(0.5)^2 \sigma_b^2 + (0.5)^2 \sigma_{SP}^2 + 2(0.5)(0.5)\rho_{b,SP}\sigma_b\sigma_{SP}}
\]

\[
= \sqrt{(0.5)^2(0.1)^2 + (0.5)^2(0.16)^2 + 2(0.5)(0.5)(0.1)(0.1)(0.16)}
\]

\[
= 0.098.
\]

Therefore, you obtain both a higher expected return and a lower standard deviation.

7. (a) CML: \( 0.25 = 0.05 + \frac{[0.20 - 0.05]}{\sigma_m} \) \( 0.04 \Rightarrow \sigma_m = 0.03. \)

SML: \( r_i = 0.05 + \frac{[0.20 - 0.05](0.5)(0.02)(0.03)}{(0.03)^2} = 10\%. \)

(b) We can use the SML to determine the \( \beta \) of the portfolio.

\[
r_i = r_f + (r_m - r_f) \beta_i \Rightarrow 0.20 = 0.05 + (0.15 - 0.05) \beta_i \Rightarrow \beta_i = 1.5.
\]

Also, when there is a riskless asset, an efficient portfolio lies on the CML. Hence

\[
r_i = r_f + \left[ \frac{r_m - r_f}{\sigma_m} \right] \sigma_i \Rightarrow 0.20 = 0.05 + \left[ \frac{0.15 - 0.05}{0.20} \right] \sigma_i \Rightarrow \sigma_i = 0.30.
\]
Finally, the correlation of the efficient portfolio with the market must equal 1, since any two efficient portfolios are perfectly correlated. To verify this:

\[ \rho_{im} = \frac{\sigma_{im}}{\sigma_i \sigma_m} = \frac{\sigma_{im}}{\sigma_m^2} \cdot \frac{\sigma_m}{\sigma_i} = \beta_i \cdot \frac{\sigma_m}{\sigma_i} = 1.5 \cdot \left( \frac{0.20}{0.30} \right) = 1. \]