Hedging with Futures and Options:
Supplementary Material

Global Financial Management

Fuqua School of Business
Duke University

Hedging Stock Market Risk:
S&P500 Futures Contract

- A futures contract on the S&P500 Index entitles the buyer to receive the **cash value** of the S&P 500 Index at the maturity date of the contract.
- The buyer of the futures contract does **not** receive the dividends paid on the S&P500 Index during the contract life.
- The price paid at the maturity date of the contract is determined at the time the contract is entered into. This is called the **futures price**.
- There are always four delivery months in effect at any one time.
  - March
  - June
  - September
  - December
- The closing cash value of the S&P500 Index is based on the opening prices on the third Friday of each delivery month.
Hedging Stock Market Risk: S&P500 Futures Contract (cont.)

- **Contract:** S&P500 Index Futures
- **Exchange:** Chicago Mercantile Exchange
- **Quantity:** $250 times the S&P 500 Index
- **Delivery Months:** March, June, Sept., Dec.
- **Delivery Specs:** Cash Settlement Based on the value of the S&P 500 Index at Maturity.
- **Min. Price Move:** 0.10 Index Pts. ($25 per contract).

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W.S.J. index futures prices for Feb’ 17, 1998

<table>
<thead>
<tr>
<th>INDEX</th>
<th>DJ INDUSTRIAL AVERAGE (CBOT) $10 times average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open</td>
<td>High</td>
</tr>
<tr>
<td>Mar</td>
<td>8440.0</td>
</tr>
<tr>
<td>June</td>
<td>8525.0</td>
</tr>
<tr>
<td>Sept</td>
<td>8610.0</td>
</tr>
</tbody>
</table>
| Est vol 10,000; vol Fr 8,369; open int 15,037, - 409.
| The Index: High 8446.32; Low 8368.60; Close 8398.50 +28.40.
| S&P 500 INDEX (CME) $250 times Index |
| Mar   | 101920| 103320| 101650| 102750| + 5.20| 103320| 854.40  | 388.113 |
| June  | 103950| 104370| 103470| 103830| + 5.30| 104370| 864.25  | 15.340  |
| Sept  | 105350| 105350| 104530| 104860| + 5.30| 105350| 884.00  | 3.933   |
| Dec   | 106180| 106400| 105550| 105890| + 5.40| 106400| 890.85  | 2.139   |
| Mr99  | ...   | ...   | ...   | 105940| + 5.00| 107180| 902.85  | 101     |
| June  | ...   | ...   | ...   | 108010| + 4.40| 108115| 959.25  | 220     |
| Dec   | ...   | ...   | ...   | 110290| + 4.50| 108840| 101490  | 334     |
| Est vol 98,041; vol Fr 66,977; open int 410,209, - 2,818.

Index prelim High 1028.02; Law 1020.09; Close 1022.78 +2.67
Valuation of the S&P500 Futures Contract

- When you buy a futures contract on the S&P500 Index, your payoff at the maturity date, $T$, is the difference between the cash value of the index, $S_T$, and the futures price, $F$.
  \[
  \text{Payoff} = S_T - F
  \]
- The amount you put up today to buy the futures contract is zero. This means that the present value of the futures contract must also be zero:
  \[
  PV(S_T - F) = 0 \Rightarrow PV(S_T) = PV(F)
  \]
- The present value of $S_T$ and $F$ is:
  \[
  PV(S_T) = S_0 e^{-dT}
  \]
  \[
  PV(F) = F e^{-rT}
  \]
- Then, using the fact that $PV(F) = PV(S_T)$:
  \[
  F = S_0 e^{(r-d)T}
  \]

Example

- On Thursday January 22, 1997 we observed:
  - The closing price for the S&P500 Index was 786.23.
  - The yield on a T-bill maturing in 26 weeks was 5.11%
  - Assume the annual dividend yield on the S&P500 Index is 1.1% per year,
    - What is the futures price for the futures contracts maturing in March, June, September, December 1997?
Example

- Days to maturity
  - June contract: 148 days
- Estimated futures prices:
  - For the June contract:
    \[ F_{June} = S_0 e^{(r-d)T} \]
    \[ = 786.23 e^{(0.0511-0.011)(148/365)} = 799.12 \]
  - Similarly:

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Days</th>
<th>Price</th>
<th>Actual Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>March</td>
<td>57</td>
<td>791.17</td>
<td>791.6</td>
</tr>
<tr>
<td>June</td>
<td>148</td>
<td>799.12</td>
<td>799.0</td>
</tr>
<tr>
<td>September</td>
<td>239</td>
<td>807.15</td>
<td>806.8</td>
</tr>
<tr>
<td>December</td>
<td>330</td>
<td>815.26</td>
<td>814.8</td>
</tr>
</tbody>
</table>

Index Arbitrage

- Suppose you observe a price of 820 for the June 1997 futures contract. How could you profit from this price discrepancy?

- We want to avoid all risk in the process.

- Buy low and sell high:
  - Borrow enough money to buy the index today and immediately sell a June futures contract at a price of 820.
  - At maturity, settle up on the futures contract and repay your loan.

<table>
<thead>
<tr>
<th>Position</th>
<th>( 0 )</th>
<th>( T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Borrow</td>
<td>782.73</td>
<td>-799.12</td>
</tr>
<tr>
<td>Buy ( e^{-dT} ) units of index</td>
<td>-782.73</td>
<td>ST</td>
</tr>
<tr>
<td>Sell 1 futures contract</td>
<td>0.00</td>
<td>820-ST</td>
</tr>
<tr>
<td>Net position</td>
<td>0.00</td>
<td>20.88</td>
</tr>
</tbody>
</table>
Index Arbitrage

Suppose the futures price for the September contract was 790. How could you profit from this price discrepancy?

Buy Low and Sell High:
» Sell the index short and use the proceeds to invest in a T-bill. At the same time, buy a September futures contract at a price of 790.
» At settlement, cover your short position and settle your futures position.

<table>
<thead>
<tr>
<th>Position</th>
<th>0</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lend e(-dT) units of index</td>
<td>-780.59</td>
<td>807.15</td>
</tr>
<tr>
<td>Sell e(-dT) units of index</td>
<td>780.59</td>
<td>-ST</td>
</tr>
<tr>
<td>Buy 1 futures contract</td>
<td>0</td>
<td>ST-790</td>
</tr>
<tr>
<td>Net position</td>
<td>0.00</td>
<td>17.15</td>
</tr>
</tbody>
</table>

Hedging with S&P500 Futures

Suppose a portfolio manager holds a portfolio that mimics the S&P500 Index.
» Current worth: $99.845 million, up 20% through mid-November ’95
» S&P500 Index currently at 644.00
» December S&P500 futures price is 645.00.
   – How can the fund manager hedge against further market movements?

Lock in a price of 645.00 for the S&P500 Index by selling S&P500 futures contracts.
» Lock in a total value for the portfolio of:
   $99.845(645.00/644.00) million = $100.00 million.

Since one futures contract is worth $250(645.00) = $161,250, the total number of contracts that need to be sold is:

\[
\frac{100.00\text{million}}{161.250} = 620.16
\]
Hedging with S&P500 Futures

Scenario I: Stock market falls
- Suppose the S&P500 Index falls to 635.00 at the maturity date of the futures contract.
- The value of the stock portfolio is: $99.845(635.00/644.00) = 98.45 million
- The profit on the 620 futures contracts is: $620(250)(645.00-635) = 1.55 million
- The total value of the portfolio at maturity is $100 million.

Scenario II: Stock market rises
- Suppose the S&P500 Index increases to 655.00 at the maturity date of the futures contract.
- The value of the stock portfolio is: $99.845(655.00/644.00) = 101.55 million
- The loss on the 620 futures contracts is: $620(250)(645.00-655) = -1.55 million
- The total value of the portfolio at maturity is $100 million.

Hedging with S&P500 Options

- Reconsider the case of a fund manager who wishes to insure his portfolio
  » holds a portfolio that mimics the S&P500 Index.
  » Current worth: $99.845 million, up 20% through mid-November ‘95
  » S&P500 Index currently at 644.00
- Lock in 645 for the S&P500 index by buying the put options at a strike price of 645, maturing in December
  » Black -Scholes value for put option is 14.96
  » Premium for one option contract is $500*14.96=$7479
  » Need to buy 310 options for portfolio of $100m:
    $310*$7479=$2.32m
    (if you borrow this now, repay $2.33 in December)
### Hedging with S&P500 Options

**Scenario I: Stock market falls**
- Suppose the S&P500 Index **falls** to 635.00 at the maturity date of the option.
- The value of the stock portfolio is: 
  \[ 99.845 \times \frac{635.00}{644.00} = 98.45 \text{ million} \]
- The **profit** on the 310 put options is: 
  \[ 310 \times 500 \times (645-635) = 1.55 \text{m} \]
- The total value of the portfolio at maturity is 
  \[ 98.45 \text{m} + 1.55 \text{m} - 2.33 \text{m} = 97.67 \text{m} \]

**Scenario II: Stock market rises**
- Suppose the S&P500 Index **increases** to 665.00 at the maturity date of the put option.
- The value of the stock portfolio is: 
  \[ 99.845 \times \frac{665.00}{644.00} = 103.1 \text{m} \]
- The **put** remains unexercised in this case
- The total value of the portfolio at maturity is 
  \[ 103.1 \text{m} - 2.33 \text{m} = 100.77 \text{m} \]

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### Hedging Interest Rate Risk With Futures Contracts

- There are two main interest rate futures contracts:
  - Eurodollar futures (CME)
  - US Treasury-bond futures (CBOT)

- The Eurodollar futures is the most popular and active contract. Open interest is in excess of $4 trillion at any point in time.
The Eurodollar futures contract is based on the interest rate payable on a Eurodollar time deposit.

This rate is known as **LIBOR** (London Interbank Offer Rate) and has become the benchmark short-term interest rate for many US borrowers and lenders.

Eurodollar time deposits are non-negotiable, fixed rate US dollar deposits in offshore banks (i.e., those not subject to US banking regulations).

US banks commonly charge LIBOR plus a certain number of basis points on their floating rate loans.

LIBOR is an annualized rate based on a 360-day year.

Example: The 90-day LIBOR 8% interest on $1 million is calculated as follows:

\[
\frac{0.08}{4} \times ($1,000,000) = $20,000
\]
Eurodollar Futures Contract

- The Eurodollar futures contract is the most widely traded short-term interest rate futures.
- It is based upon a 90-day $1 million Eurodollar time deposit.
- It is settled in cash.

- **At expiration**, the futures price is 100-LIBOR.
- **Prior to expiration**, the quoted futures price implies a LIBOR rate of:

  \[
  \text{Implied LIBOR} = 100 - \text{Quoted Futures Price}
  \]

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Eurodollar Futures Contract

- **Contract**: Eurodollar Time Deposit
- **Exchange**: Chicago Merchantile Exchange
- **Quantity**: $1 Million
- **Delivery Months**: March, June, Sept., and Dec.
- **Delivery Specs**: Cash Settlement Based on 3-Month LIBOR
- **Min Price Move**: $25 Per Contract (1 Basis Pt.)

\[
\frac{(1/100)(1\%)(1,000,000)}{4} = $25
\]
Suppose in February you buy a March Eurodollar futures contract. The quoted futures price at the time you enter into the contract is 94.86.

If the 90-day LIBOR rate at the end of March turns out to be 4.14% p.a., what is the payoff on your futures contract?

- The price at the time the contract is purchased is 94.86.
- The LIBOR rate at the time the contract expires is 4.14%. This means that the futures price at maturity is 100 - 4.14 = 95.86.

In dollar terms, our payoff is:

\[ Payoff = \frac{(95.86 - 94.86)(10,000)}{4} = 2,500 \]

- The increase in the futures price is multiplied by $10,000 because the futures price is per $100 and the contract is for $1,000,000.
- We divide the increase in the futures price by 4 because the contract is a 90-day (90/360) contract.
Hedging Interest Rate Risk With Futures Contracts

- Suppose a firm knows in February that it will be required to borrow $1 million in March for a period of 90 days.

- The rate that the firm will pay for its borrowing is LIBOR + 50 basis points.

- The firm is concerned that interest rates may rise before March and would like to hedge this risk.

- Assume that the March Eurodollar futures price is 94.86.

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Hedging Interest Rate Risk with Futures (cont.)

- Step 1: Specify the risk.
  - Your company will lose if interest rates rise. That is, if the interest rate is higher, your firm will have to pay more interest on the loan.

- Step 2: Determine an appropriate futures position.
  - You want a futures position that gives a positive return if interest rates rise. That is, you want a futures position that gives a positive return if (100-LIBOR) falls. Hence, you want a futures position that gives a positive return if the futures price falls. Therefore you sell Eurodollar futures.

- Step 3: Determine the amount.
  - $1 mm amounts to one contract.
Hedging Interest Rate Risk
With Futures (cont.)

- The LIBOR rate implied by the current futures price is:
  \[ 100 - 94.86 = 5.14\%. \]

- If the LIBOR rate increases, the futures price will fall. Therefore, to hedge the interest rate risk, the firm should sell one March Eurodollar futures contract.

- The gain (loss) on the futures contract should exactly offset any increase (decrease) in the firm’s interest expense.

Hedging Interest Rate Risk
With Futures (Cont.)

- Suppose Libor increases to 6.14% at the maturity date of the futures contract.

- The interest expense on the firm’s $1 million loan commencing in March will be:
  \[
  \frac{- (0.0614 + 0.005)(1,000,000)}{4} = -16,600
  \]

- The payoff on the Eurodollar futures contract is:
  \[
  \frac{- (93.86 - 94.86)(10,000)}{4} = 2,500
  \]
Hedging Interest Rate Risk with Futures (Cont.)

- LIBOR Rate is 6.14%.

<table>
<thead>
<tr>
<th>Cash Flow</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest on Loan</td>
<td>-16,600</td>
</tr>
<tr>
<td>Futures Payoff</td>
<td>2,500</td>
</tr>
<tr>
<td>Net Payoff</td>
<td>-14,100</td>
</tr>
</tbody>
</table>

Now assume that the LIBOR rate falls to 4.14% at the maturity date of the contract.

- The interest expense on the firm’s $1 million loan commencing in March will be:
  \[
  -\frac{(0.0414 + 0.005)(1,000,000)}{4} = -11,600
  \]

- The payoff on the Eurodollar futures contract is:
  \[
  -\frac{(95.86 - 94.86)(10,000)}{4} = -2,500
  \]
Hedging Interest Rate Risk with Futures (Cont.)

- LIBOR Rate is: 4.14%.

<table>
<thead>
<tr>
<th>Cash Flow</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest on Loan</td>
<td>-11,600</td>
</tr>
<tr>
<td>Futures Payoff</td>
<td>-2,500</td>
</tr>
<tr>
<td>Net Payoff</td>
<td>-14,100</td>
</tr>
</tbody>
</table>

The net outlay is equal to $14,100 regardless of what happens to LIBOR.

- This is equivalent to paying 5.64% p.a. over 90 days on $1 million.

- The 5.64% borrowing rate is equal to the current implied LIBOR rate of 5.14%, plus the additional 50 basis points that the firm pays on its short-term borrowing.

- The firm’s futures position has locked in the current implied LIBOR rate.