Solutions to Practice Questions

(Forwards and Futures)

1. These practice questions are a supplement to the problem sets, and are intended for those of you who want more practice. They are Optional, and are not part of the required material.

2. It is recommended that you look at these problems only after you fully understand how to solve the problem sets, the examples we covered in class, and the ones in the lecture notes.

3. Please note that I have collected these examples from previous teaching material I have had. As such, while in most cases the notation will match the one used in class, the match is not 100%.

4. Some of these questions are easier than the ones you are expected to know how to solve, while others are above the level of knowledge you are expected to show on quizzes and the final.

ENJOY!
1. (a) The price of the three month future is

\[ F_t = S_t(1 + r)^{T-t} = 80 \times 1.01^3 = 82.42 \]

(b) The fair value of the 6 month future should be

\[ F_t = S_t(1 + r)^{T-t} = 80 \times 1.01^6 = 84.92. \]

Given the trading price $90, we know that it is overvalued. Hence, to make arbitrage profit, we want to short the future.

At \( t=0 \), we short one unit of the future; borrow $80 to buy one unit of the stock at spot market. The net cash flow is zero.

At \( t=1 \), we deliver the unit to the buyer, receive payment of $90 on the future, and repay the bank loan \( 80 \times 1.01^6 = 84.92 \). The net cash flow is \( 90 - 84.92 = 5.08 \). This is our arbitrage profit.

(c) The fair value of the 1 year future should be

\[ F_t = S_t(1 + r)^{T-t} = 80 \times 1.01^{12} = 90.146. \]

Given the trading price $90, we know it is undervalued. Hence, to make arbitrage profit, we want to long the future.

At \( t=0 \), we long one unit of the future, borrow and short one unit of the stock at spot market, and save the $80 proceeds from stock selling in bank. The net cash flow is zero.

At \( t=1 \), we acquire one unit of the stock from the seller of the future and return the stock to lender. We have to pay $90 according to the future contract. On the other hand, we withdraw from the bank \( 80 \times 1.01^{12} = 90.146 \). The net cash flow \( 0.146 \) is our arbitrage profit.

2. The quarterly interest rate is \( 1.28\%/4 = 0.32\% \) on USD and \( 2.52\%/4 = 0.62\% \) on Euro. We construct a replicating strategy to obtain 1 Euro in three months: we buy \( 1/(1 + 0.62\%) \) Euro today and deposit it in a bank. We borrow sufficient USD today to purchase this amount of Euro, and repay the debt in three months. The repayment is the cost (in three months) of one Euro (in three months). Hence, the future cost will be, in USD,

\[
1/(1 + 0.62\%)/0.9250 \times (1 + 0.32\%) = 1.07786
\]

In other words, the forward rate would be 1 Euro @ 1.07786 USD or 1 USD @ 0.9278 Euro.

3. The 3 month futures price implies that spot gold should be trading at $306/1.02 = $300.

The 6 month futures price implies that spot gold should be trading at $318/1.05 = $302.857.

Thus the 6 month contract is overpriced relative to the 3 month contract.

- Underline

  - Go long the 3 month contract.
  - Go short the 6 month contract.
  - Lend $300 for 3 months.
  - Borrow $302.857 for 6 months.

Net $2.857 = arbitrage profit.
• At the end of 3 months:
  ◦ Take delivery of gold.
  ◦ Pay $306 with proceeds of maturing T-Bills.
    Store gold for one quarter.
• At the end of 6 months:
  ◦ Deliver the stored gold to close out your short futures position.
  ◦ Use the $318 futures price received upon delivery to repay your borrowing.

4. In the equality, Futures Price = Spot Price + Costs of Carry − Benefits of Carry, the (Benefits of Carry) are the benefits to a party just indifferent to adding an additional unit of copper to his/her inventory throughout the life of the contract. Suppose that today you go long the June contract. Suppose that in June you borrow to pay the futures price, and you add the copper you receive to your inventory.

Suppose also that today you go short in the September contract.

Your payoff in September when you deliver the copper to close out your short futures position will be

\[
(1.056 - 1.0725 \times 1.01646 - 3 \times \frac{1.95}{2000} + \text{Convenience Value}) \text{ per lb.} \\
= -3.71\varphi \text{ per lb.} + \text{Convenience Value.}
\]

You would have to place a value (measured in September dollars) on the flexibility associated with having an additional unit of copper in inventory between June and September of at least 3.71\varphi per lb. before the strategy would be attractive.