Graphical Approach to Forward Contracts

In this note we examine the relationship between forward contracts, bonds and the underlying asset. For simplicity we use the example given in class where we derived the price of a forward on a non-dividend paying stock. This stock trades today for $25 and we consider a forward contact that expires in 3 months from now ("the maturity").

We begin by plotting *payoff diagrams* for various assets. These diagrams show the *payoff to the owner of the asset at maturity*. These payoffs do not include any costs or gains earned when purchasing the assets today.

**Long/Short the security:**

![Payoff to long and short positions in the stock](image)

**Long/Short Forward:**

![Payoff to long and short position in Forward Contract](image)

Note that both the long and short forward payoff positions break even when the price of the stock at maturity is equal to the forward price (25.375 in our example).
Buy/sell a bond for $25 with 1.5% quarterly return:

By buying a bond (lending) today we know that we are going to get a fixed payoff equal to $25 \times 1.015 = 25.375$. By selling a bond today (borrowing) we know that we are committed to repay 25.375.

The previous plots enable us to achieve the following goals:

1. Construct a forward contract using only the bond and stock.
2. Construct a stock payoff using only the forward contract and the bond.
3. Construct a bond payoff using only the forward contract and the stock.

1. Construction of a long forward contract using the stock and bond:

The payoff of the long forward can be replicated by borrowing $25 and buying the stock. At maturity the payoff is just the sum of the payoffs of the constituent assets:
The payoff diagram when we sell a forward contract can be obtained by reversing the above actions.

2. Construction of a long stock payoff using the forward contract and the bond.

The payoff of the long stock can be replicated by lending $25 and entering into a long forward position. Again, at maturity the payoff is just the sum of the payoffs of the constituent assets:

The payoff diagram of a short stock position can be obtained by reversing the above actions.

3. Constructing a long bond payoff (namely the position of an investor who lent $25) using the forward contract and the stock:

The payoff of a short bond (borrowing) can be obtained by reversing the above actions.