Relative Wealth Concerns and Financial Bubbles

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We present a rational general equilibrium model that highlights the fact that relative wealth concerns can play a role in explaining financial bubbles. We consider a finite-horizon overlapping generations model in which agents care only about their consumption. Though the horizon is finite, competition over future investment opportunities makes agents' utilities dependent on the wealth of their cohort and induces relative wealth concerns. Agents herd into risky securities and drive down their expected return. Even though the bubble is likely to burst and lead to a substantial loss, agents' relative wealth concerns make them afraid to trade against the crowd. (*JEL* G11, G12, D52, D53, D91, E43)

In this article we present a stylized model that highlights the fact that relative wealth concerns may play an important role in explaining the presence and dynamics of financial "bubbles" and excess volatility. The role of relative wealth concerns in explaining these phenomena is quite simple. In standard models, rational agents exploit price anomalies by selling overpriced assets and buying undervalued assets. This activity of "buy low/sell high" eliminates price distortions in equilibrium. However, if agents are sensitive to the wealth of others, trading against the crowd increases the risk of their relative wealth. As a result, rational traders may sustain prices that are too high even though they understand that these prices deviate substantially from fundamentals.

Thus, relative wealth concerns help support the existence of financial bubbles by increasing the risk of trading against the crowd. We show in

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addition that when relative wealth concerns are sufficiently strong, they can promote the *creation* of price bubbles. In this case, starting from optimal risk sharing, a small deviation by one agent may induce other agents to deviate such that the aggregate response leads to an amplification of the original deviation. As a result, optimal risk sharing will not be stable, and asset price distortions will emerge in all stable equilibria.

While the standard assumption in economic models is that utility is derived from the absolute level of one's own consumption, economists have long believed that relative considerations are important. Indeed, Veblen (1899) argued that as society becomes richer, the amount of consumption necessary to maintain one's social standing—an important component of utility—increases. Frank (1985) has also emphasized the importance of relative wealth in determining social status. More recently, such features have been incorporated in asset pricing models such as Abel (1990) "catching up with the Joneses" utility specification.[1] All these papers assume that utility functions are such that other people's wealth or consumption levels impact one's own utility through an exogenous dependence of the utility function on relative wealth.

One limitation of assuming an exogenous preference for relative wealth is that choosing the functional form provides many degrees of freedom. We show that the relative wealth effects that are necessary to create and sustain price bubbles can also arise endogenously in a fully rational model in which agents care only about their own consumption. In particular, if there are scarce goods whose prices increase with the wealth of investors, then agents' abilities to consume will depend on their relative wealth. These relative wealth concerns can induce herding, which has an aggregate impact on equilibrium prices.[2]

We consider a simple finite horizon overlapping generations model.[3] In this model, the "scarce goods" are future investment opportunities. When the wealth of middle-age investors is high, competition for these investments drives down equilibrium returns, raising the cost of funding their retirement. Thus, relative wealth concerns emerge for these agents. As a result, when agents are young they will wish to invest in assets that are

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1 See also Gali (1994); Bakshi and Chen (1996); Abel (1999); Chan and Kogan (2002), and Gomez et al. (2006).

2 Cole, Mailath, and Postlewaite (1992, 2001) develop an alternative model of endogenous relative wealth effects. In their models, individuals compete for mates, and their success depends upon their relative wealth. This non-market competition over mates has a similar effect as the market competition over scarce resources that occur in our model. They consider the implications of relative wealth concerns for portfolio choice and investment rates. Becker, Murphy, and Werning (2005) develop a model of relative wealth effects based on individual preferences for status that is acquired through purchases of a "status good." They assume that status increases an agent's marginal utility of consumption, and show how this leads to excessive risk taking in equilibrium. Neither paper considers the effect on asset price dynamics.

3 We choose a finite horizon to avoid the well-known possibility of bubbles in infinite-horizon OLG models, which occur for a much different reason.

highly correlated with the investments of others in their cohort. This desire can lead agents to herd into an asset class, raising its price and lowering its future return.

The model is presented in Section 2. It is standard with the following features. First, there is no information asymmetry and the horizon is finite. Second, agents are fully rational and maximize a constant relative risk aversion (CRRA) utility function over their own consumption. Third, agents trade two financial assets: a risk-free bond that pays one unit each period, and a risky security that pays either two units or zero units with equal likelihood each period.

When there is no aggregate risk, there is a natural and obvious equilibrium for the above setting: the risky security is not traded, and has a price equal to the price of the risk-free bond. Indeed, in a standard complete markets model this price is the only price consistent with the absence of aggregate risk in this economy, and we therefore refer to it as the "fundamental" price. One might expect that this would be the unique equilibrium in this setting.

We show, however, that bubble-like price dynamics can arise in equilibrium in this economy. Specifically, not only can prices of the risky security deviate substantially from their fundamental value, but as long as a deviation persists it is also amplified over time until it suddenly collapses. These price dynamics occur despite the fact that the horizon is finite and agents are rational and use backward induction in solving their respective portfolio choice problems. In a striking representative example of the equilibrium we present, the price of the risky security rises to an 80% premium over the price of the risk-free annuity while it continues to pay dividends, with this premium vanishing as soon as a dividend is missed.

As agents in our model are completely rational, one could argue that the equilibrium we describe is not a "bubble." The literature does not have a clear definition for this term. In his book Cochrane (2001) writes: "*I close with a warning: The word 'bubble' is widely used to mean very different things. Some people seem to mean any large movement in prices. Others mean large movement in prices that do correspond to low or perhaps negative expected returns.*" In order to fix ideas, we define a bubble for an economy with risk-averse agents as an equilibrium in which (1) an asset with cash flows that have a nonnegative correlation with aggregate risk trades at a price that exceeds the expected present value of its cash flows when evaluated at the risk-free rate; and (2) individuals rationally choose to hold the asset, despite their knowledge of (1).

Condition (1) implies a negative risk premium for the asset, distinguishing a bubble from risk tolerance. Condition (2) highlights the fact that agents are aware of the asset's low return. This distinguishes a bubble from a simple mistake due to incorrect expectations or partial information.

We should note that some define a bubble as a case in which an asset that pays no dividends trades with a positive price (i.e. fiat money *a la* Samuelson (1958)).[4] While that cannot occur in our model, we can obtain price patterns that approximate this behavior, as the probability that the price will eventually drop substantially can be made arbitrarily close to 1.

As we demonstrate in Section 2, the driving force behind this seemingly odd behavior is endogenous relative wealth concerns. Agents realize that the wealth of their cohort will drive up future asset prices and thus lower their returns. Therefore, to meet any given level of retirement income, agents need to save more when their cohort is wealthy. This externality induces a herding incentive: agents choose to imitate the portfolio choices of their cohort to avoid being poor when their cohort is wealthy. As a result, young investors may herd into the risky asset, driving up its price. The price distortion grows over time since as the young become wealthier, their impact on asset prices grows, further strengthening the herding incentive.

We also show that when there is aggregate risk in the economy (so that the risky security is positively correlated with aggregate risk), as long as agents are more risk averse than log utility, there is always an equilibrium in which the young investors herd and overinvest in the risky security. This overinvestment raises the price and drives down the risk premium of the risky security, to the point that for typical parameters the risk premium becomes negative.

Section 3 demonstrates how these factors may even affect a simple deterministic saving/consumption pattern in a finite-horizon overlapping generation model. We choose a constant endowment structure so that there exists a simple benchmark equilibrium in which agents' consumption is one unit per period, and in which the interest rate is equal to zero. However, we show that there exists another equilibrium in which interest rates differ from zero, and this difference initially increases over time. As with the stochastic case, prices deviate from fundamentals because of endogenous relative wealth concerns, even though agents are rational and the horizon of the economy is finite. Furthermore, as we add more periods, the equilibrium outcome converges to a steady-state equilibrium of an infinite-horizon OLG model. The prices in the steady state deviate from the benchmark but this is not due to a fiat money effect. As our analysis demonstrates, this equilibrium is the limit of equilibria in models with a finite horizon.

In our model, relative wealth concerns arise endogenously as a result of a pecuniary externality. In Section 4 we relate our model to those that inject relative *consumption* considerations directly into the utility function.

[4] See Tirole (1985) who follows this definition.

This captures the fact that agents care directly about what others consume. We refer to this approach as an *exogenous* specification of relative utility.

With either exogenous or endogenous relative utility, we analyze the conditions that are required for bubble-like behavior to arise in equilibrium. Our analysis emphasizes a general equilibrium approach. We show that there are two key requirements. First, relative wealth concerns must be sufficiently strong. Second, there must be sufficient heterogeneity across agents' preferences. If agents are identical, a representative agent characterization will prevail and prices will be determined by aggregate fundamentals. In our setting, agents can be separated into distinct *communities* based on their preferences; in the overlapping generations setting a community is an age cohort. Because of different herding incentives, these communities trade with each other, breaking the link between aggregate consumption and asset prices. However, we show that the "keeping up with the Joneses" property of preferences alone are not sufficient to sustain herding in equilibrium, and derive a more stringent necessary condition in Section 4.

1. Related Literature

This paper is related to several strands of the literature. First the paper is related to the literature that examines relative wealth concerns (e.g., Frank (1985)). In the finance literature, Abel (1990) was the first to introduce relative considerations in his paper on "catching up with the Joneses." His exogenous preferences approach was the basis for Gali (1994), who analyzes the potential impact of consumption externalities on the equity risk premium. Bakshi and Chen (1996) show that when investors care about other investors wealth prices can be more volatile. Our approach is somewhat closer to Gali (1994) and Bakshi and Chen (1996) than to Abel (1990), as agents care about wealth (consumption) relative to current, as opposed to lagged, aggregate wealth (consumption) per capita. We differ from this literature in that we examine how relative considerations can arise endogenously, and we focus on bubble-like behavior in financial markets.

The mechanism leading to herding in our model differs from papers in which herding incentives in investment are an outcome of managers' concerns about their reputation. For example, Scharfstein and Stein (1990) show that when a manager's ability is *ex ante* unknown, then he may choose to follow the actions of other managers.

There is an extensive literature on bubbles in financial markets. We clearly do not have the space here to survey this vast literature (see Brunnermeier (2001) for an excellent survey). Instead we just note that our model is fully rational, with prices being endogenous as they are obtained by crossing demand and supply in the usual way. We do not rely on an infinite horizon and/or "fiat money." While Allen and Gale (2000), and Allen, Morris, and Postlewaite (1993) describe finite-horizon models in which bubbles occur, these models rely on asymmetric information and short sales or liquidity constraints. In our model, all agents are symmetrically informed, and short sales are unrestricted.

The one key imperfection in our framework is the fact that markets are incomplete. From a technical perspective our analysis is therefore related to the classic literature on incomplete markets. This literature has shown that with incomplete markets and multiple goods, one could expect multiple equilibria, some of which are not even constrained efficient [see, for example, Hart (1975); Stiglitz (1982), and Geanakoplos and Polemarchakis (1986)]. We note, however, that the incompleteness in our model arises from limited participation (unborn agents cannot trade) rather than an absence of securities. More closely related are models of extrinsic uncertainty, or sunspots, introduced by Cass and Shell. The equilibrium we describe in Section 2 in the absence of aggregate risk is an example of what Cass and Shell (1983) call a "sunspot equilibrium." This is because prices and the allocation of resources depend on a purely extrinsic random variable—the payoff of the risky security is unrelated to fundamentals.[5] Our results are distinct from this literature, however. In both the setting with aggregate risk in Section 2, and in the purely deterministic setting of Section 3, our equilibria are no longer sunspots. Also, in our analysis, we focus on the characterization of the price dynamics, as opposed to existence results. Finally, we demonstrate the link between relative wealth concerns and potential sunspots, and show how herding incentives can create price bubbles in a finite and standard economy.

Gomez, Priestley, and Zapatero (2006) develop and test an international asset pricing model in which agents care about the contemporaneous average consumption in their country. They are the first to demonstrate empirically that such preferences can lead to negative risk premium for the domestic risk factor, which is similar to the bubble-like distortions identified here. They further show that their exogenous preference model yields similar predictions to an endogenous specification, such as in our article. Importantly, using cross-country data they find significant empirical support for these effects, showing that negative domestic risk premia arise in an international multifactor asset pricing model. Here we demonstrate how these effects may produce bubble-like price dynamics.

Finally, our article is related to DeMarzo, Kaniel, and Kremer (2004). The important difference is that the focus here is on price effects rather than portfolio choices. In DeMarzo, Kaniel, and Kremer (2004), we eliminated

[5] One distinction of our example from the typical construction of a sunspot equilibrium is that ours does not rely on randomization over multiple equilibria in the subgame. (Though, see the appendix of Cass and Shell (1983) for another example that is not a mere randomization).

price effects by assuming that communities are symmetric. In addition, in that paper only a static model is considered. The reason for this focus was the desire to obtain a simple closed-form solution and concentrate on the implications for portfolio choice. Here we analyze a dynamic model in which we can examine price effects and bubble-like behavior, though the cost is that we must rely in many cases on numerical solutions.

2. A Model of Speculative Bubbles

We describe a model in which agents support prices that deviate substantially from "aggregate fundamentals" despite the fact that they know the bubble is unlikely to persist. Two important features of our model are that it has a finite horizon and that there is no asymmetric information.

We use a variant of a finite overlapping generations model for our analysis. There are three dates (0, 1, 2) and three types of agents: Middle-aged (M) agents live at dates 0 and 1, Young (Y) agents live at dates 0, 1, and 2, and Unborn agents (U) live at dates 1 and 2. Table 1 shows the consumption good endowments of the different agent types, where "—" specifies that an agent type does not exist during a period and "0" specifies that the type exists but has no endowment. For simplicity, we assume that date 0 consumption has already occurred, so that there is no endowment at that date.[6]

All agents maximize a time separable CRRA utility function over consumption in dates $t = 1,2$, where we simplify the analysis by assuming no discounting. Young and unborn agents maximize $(1-\gamma)^{-1}E(c_1^{1-\gamma} + c_2^{1-\gamma})$, where c_t denotes consumption at date t. Middle-aged agents, who consume only at date 1, maximize $(1-\gamma)^{-1}E(c_1^{1-\gamma})$. To highlight the role of relative wealth considerations, we first assume all endowments are riskless. We will consider aggregate endowment risk later in the section.

To introduce the possibility of speculative trade, we allow agents to trade two short-lived securities, both in zero net supply. Suppose that the probability of rain in each period is 50%. Each period, agents may trade:

- a risky security that pays two units of consumption next period if it rains and zero if it does not;
- a risk-free bond that pays one unit of consumption next period with certainty.

With riskless endowments and risk-averse agents, the standard equilibrium analyzed in this setting is as follows: At date 0, young and middle-aged agents do not trade at all, and at date 1, young and unborn

[6] The analysis would be similar, though more complicated, if we included date 0 consumption as well.

Table 1
Endowments of middle-aged, young and unborn agents

Date	0	1	2
Middle-aged	0	e_{M1}	—
Young	0	e_{Y1}	0
Unborn	—	0	e_{U2}

agents trade the risk-free bond to smooth consumption. Because agents hold no risk, the risky security will not command a risk premium. Because the expected payoff of the risky security is the same as the payoff of risk-free bond, both securities will have the same equilibrium price.

As our analysis will show, however, even with riskless endowments equilibria exist in which speculative trade occurs at date 0 between young and middle-aged agents, with these agents taking on non-fundamental risk.[7] This trade is motivated by the young agents' concern over the competition they will face at date 2. Speculative purchases of the risky security by the young agents will raise its price substantially above the price of the risk-free bond, implying a negative risk premium. We initially consider the basic setting with only a single round of speculative trading between date 0 and date 1. Having established the key insights for that simpler case, we then allow multiple rounds of speculative trade and consider the resulting price dynamics.

The fact that a speculative bubble arises in our model depends on the fact that the markets are incomplete. The incompleteness is *not* due to a missing asset but due to unborn agents being unable to trade at time zero, prior to being born. Without incompleteness the first welfare theorem would apply, and we would not get an inefficient outcome. If the unborn agents were able to trade at time zero, they would make offsetting trades that would eliminate the possibility of an equilibrium with speculative trading.

2.1 A single round of speculative trade

In our model, speculative trade results from relative wealth concerns. It is the young agents in our model who are subject to these concerns. Young agents know that at date 1 they will engage in nonspeculative trade with the now unborn agents, exchanging date 1 consumption for date 2 consumption. However, they will be competing with other agents in their cohort to conduct this trade. Thus, the price at which they can make this exchange—that is, the equilibrium interest rate—will be determined by the wealth of their cohort.

[7] Note that with risk-free endowments, any stochastic payoff is unrelated to aggregate fundamentals. We begin with an economy without aggregate risk to make it easiest to see our results. But we will show later in this section that with aggregate risk the equilibrium we describe will no longer be a "sunspot," yet will still behave as a bubble.

2.1.1 Date 1 trading. Middle-aged agents, at the end of their life, will simply consume all their wealth. Thus, between dates 1 and 2, the model is a standard two-date exchange economy involving the young and unborn agents. As such, standard results imply that no speculative trade will occur, and trading between young and unborn agents utilizes only the risk-free asset.

With CRRA utility, young and unborn agents' consumption across the two dates will be proportional to each other. The (gross) equilibrium interest rate R_1 at date 1 will be determined by the aggregate equilibrium holdings of date 1 consumption goods by the young generation, which we denote by W_{Y1}, and the date 2 endowment of consumption goods (e_{U2}) of the unborn agents:

$$R_1 = \left(\frac{e_{U2}}{W_{Y1}}\right)^{\gamma} \tag{1}$$

That is, the equilibrium interest rate at date 1 is increasing in the aggregate consumption growth rate of the young and unborn agents. A standard derivation shows that given R_1, the indirect utility of a young agent with wealth w (measured in date 1 consumption units) is given by:

$$\frac{1}{1-\gamma} w^{1-\gamma} \left(1 + R_1^{(1-\gamma)/\gamma}\right)^{\gamma} \tag{2}$$

Naturally, this indirect utility function is increasing in R_1—a higher interest rate makes it easier for the young to save for retirement (note that when $\gamma > 1$, utility is negative).

Combining (1) and (2), we conclude that the indirect utility of a young agent is given by:

$$v(w, W_{Y1}) = \frac{1}{1-\gamma} w^{1-\gamma} \left[1 + \left(\frac{W_{Y1}}{e_{U2}}\right)^{\gamma-1}\right]^{\gamma} \tag{3}$$

If $\gamma > 1$, then the marginal indirect utility of a single young agent is increasing in the aggregate wealth of other young agents. In this case, there is an important strategic complementarity between young agents: each young agent prefers to have a positive correlation between his own wealth and the wealth of his cohort. This property is critical in generating the price dynamics we are interested in (see Section 3 for further discussion).

2.1.2 A speculative equilibrium. We can now solve for an equilibrium at date 0 in this economy as though it were a standard exchange economy between young and middle-aged agents, terminating at date 1, and with the young agents' preferences over consumption at date 1 given by the indirect utility function (3). Because there is no consumption at date 0, we let the risk-free bond be the numeraire, and let p_0 denote the price at date 0 of the risky security relative to the risk-free bond.

To derive the equilibrium, consider the optimal response of a single young agent to the aggregate actions of the other young agents. We examine the reaction to a speculative bias in the portfolio of the young cohort, which we denote by σ. That is, suppose young agents hold portfolios such that $W_{Y1} = c(1+\sigma)$ if it rains and $W_{Y1} = c(1-\sigma)$ if it does not, for some constant c. If young agents take no risk then $\sigma = 0$; if they hold a positive amount of the risky security then $\sigma > 0$. Given an aggregate bias σ of the young cohort, we compute the optimal bias for an individual young agent (assuming all other cohorts are also optimizing). We denote the optimal bias in his portfolio by $m(\sigma)$. Equilibrium then corresponds to a fixed point $m(\sigma) = \sigma$, in which it is optimal for each individual young agent to choose the same bias as his cohort.

We derive the function $m(\sigma)$ as follows. Given the aggregate trades of the young, the aggregate consumption of the middle-aged agents can be determined by market clearing. The marginal rate of substitution of these agents then determines the equilibrium price ratio, p_0, as follows:

$$\frac{p_0}{2-p_0} = \left(\frac{e_{Y1} + e_{M1} - c(1+\sigma)}{e_{Y1} + e_{M1} - c(1-\sigma)}\right)^{-\gamma} \tag{4}$$

The right-hand side of (4) is the marginal rate of substitution of the middle-aged agents between rain and no-rain states. Because buying two bonds and shorting the risky security pays two units of consumption if it does not rain, the left-hand side is the relative price of consumption across these states.

Note that purchasing the risky security and shorting the bond shift consumption from the no-rain state to the rain state. Thus, the young agents must satisfy the budget constraint:

$$e_{Y1} = c + c\sigma(p_0 - 1) \tag{5}$$

Together, Equations (4) and (5) allow us to solve for c and p_0 as a function of the aggregate bias σ of the young agents. Given the price p_0, we then solve for the optimal trade $m(\sigma)$ of an individual young agent, given the indirect utility function (3). This utility function takes into account the aggregate impact of the young agents on the interest rate that will prevail at date 1.

Despite the complexity of the above procedure, the following simple condition suffices for the existence of a speculative equilibrium:

Proposition 1. *There exists a speculative equilibrium (i.e., $m(\sigma) = \sigma$ with $\sigma > 0$ and $p_0 > 1$) if*

$$\gamma > 2 + \frac{e_{Y1}}{e_{M1}} + \left(\frac{e_{U2}}{e_{Y1}}\right)^{\gamma-1} \left(1 + \frac{e_{Y1}}{e_{M1}}\right) \tag{6}$$

Note that to satisfy (6), agents must be sufficiently risk averse. The degree of risk aversion required increases with endowment of the young relative to middle-aged, e_{Y1}/e_{M1}. Intuitively, if the middle-age community is small, the price impact of the young traders is large, making it more difficult to support a speculative equilibrium. Finally, the condition is more easily satisfied if the endowment of the unborn is low, as a decrease in e_{U2} makes the price of the date 2 consumption good (that is, the interest rate) more sensitive to the young cohort's wealth.

Figure 1 illustrates the young agents' optimal response function m for the case in which all endowments are equal to 1 and $\gamma = 6$. Also shown for reference is the optimal response m_o for the middle-aged agents who do not have relative wealth concerns. Note that they find it optimal to trade against the bias.

As Figure 1 shows, one equilibrium is the benchmark, nonspeculative equilibrium with $m(0) = 0$. In this equilibrium, there is no trade at date 0,

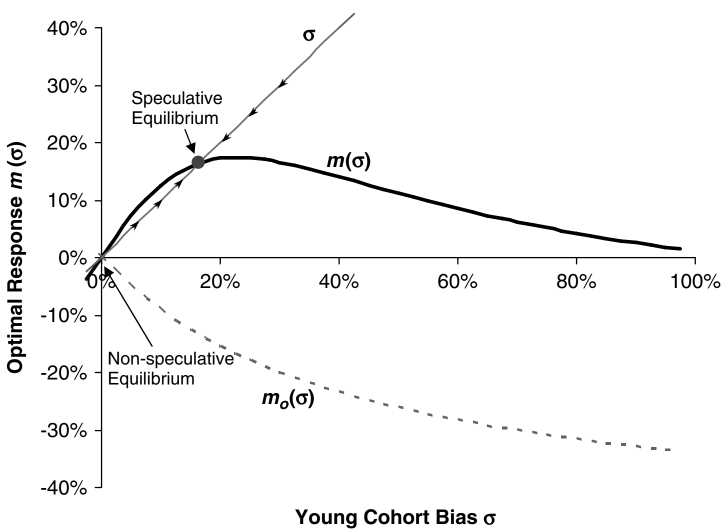

Figure 1
Optimal response $m(\sigma)$ of a Young agent if Young cohort has aggregate bias σ
Note the two equilibria corresponding to the two fixed points of m: The benchmark, nonspeculative equilibrium $\sigma = 0$, and the speculative equilibrium with $\sigma > 0$. The function m_o shows the optimal response of the middle-aged cohort, who is not subject to relative wealth concerns.

and at date 1 the young and unborn agents smooth consumption given a gross interest rate $R_1 = 1$, and so consume 0.5 each period.

There is another, speculative equilibrium with $\sigma = 16.4\%$. In this equilibrium, the relative price of the risky security is $p_0 = 1.67$.[8] The existence of the speculative equilibrium arises from the fact that for any small bias in the portfolio of the young cohort, individual young agents will optimally react with a larger bias in their own portfolios as they attempt to hedge the fluctuations in next period's interest rate.

Table 2 describes equilibrium wealth, consumption, and interest rates in the speculative equilibrium. Because the young buy the risky security, the middle-aged must sell it, reducing their consumption in the rain state. Given its negative correlation with their wealth, the middle aged will demand a high price of $p_0 = 1.67$ for the risky security. If we normalize the length of a generation to be 25 years, this price corresponds to a negative risk premium of -2.03%.

The young pay $p_0 - 1 = 0.67$ to shift one unit of wealth to the rain state, so that a bias of 16.4% reduces their expected wealth to $c = 1/[1 + 16.4\%(.67)] = 0.901$. The young agents' wealth in the rain state of $c(1 + \sigma) = 0.901(1.164) = 1.049$, relative to the unborn's endowment of 1 in date 2, drives down the gross equilibrium interest rate to $R_1 = 0.752$, which corresponds to a net annual interest rate of $r_1 = 0.752^{1/25} - 1 = -1.13\%$. Given the low interest rate in the rain state, the young must save a great deal to fund their retirement. As a result, they consume only $0.462/1.049 = 44\%$ of their wealth at date 1.

By contrast, in the no-rain state the young have wealth of only $c(1 - \sigma) = 0.753$ at date 1. But because of the high interest rate $r_1 = 7.06\%$, they can afford to consume $0.606/0.753 = 80\%$ of their wealth at date 1 and still save adequately for retirement. Thus, even though the young have higher wealth at date 1 in the rain state, because of the endogenous fluctuation in interest rates, they can afford to consume more at date 1 in the no-rain state. In fact, their consumption at date 1 is perfectly correlated with the middle-aged agents. Thus, the effect of date 1 interest rates on the young agents' ability to save for retirement explains their willingness to hold the risky security at date 0, despite its negative risk premium.

Note that in this equilibrium, speculative behavior creates endogenous risk, which then rationalizes the speculative trades. The equilibrium described above satisfies our definition of a bubble. The risky asset pays on average 1 and is uncorrelated with the aggregate endowment which is deterministic. Hence, a current price that exceeds one indicates a negative risk premium. Moreover, this negative risk premium is common knowledge. Though they are risk averse, the young agents hold the risky

[8] By symmetry, an additional speculative equilibrium exists with $\sigma = -16.4\%$. Without loss of generality Figure 1 plots only the range $\sigma > 0$.

Table 2
Wealth, consumption, and interest rates in the biased equilibrium

State	Date 1 wealth		Annual interest rate r_1 (%)	Young consumption	
	Middle-aged	Young		Date 1	Date 2
Rain	0.951	1.049	−1.13	0.462	0.441
No rain	1.247	0.753	7.06	0.606	0.806

asset because others in their cohort do so. If a young agent chose not to hold the risky asset, the agent would face the risk that others who do would drive down the equilibrium interest rate next period when the asset pays off, and the agent would face a high cost of saving for retirement.

As Figure 1 shows, relative wealth concerns induced by incomplete markets are critical for the existence of a bubble. If the model ended at date 1, the young agents' optimal response would equal that for the middle-aged agents, m_o, and the unique equilibrium would be $\sigma = 0$. Alternatively, if the model continued until date 2, but unborn agents could trade at date 0, young and unborn agents would trade at date 0 to hedge their exposure to interest rate risk, and again the unique equilibrium would be $\sigma = 0$.[9] We emphasize, however, that the nature of the incompleteness in our model is not due to missing assets. Adding arbitrary assets to the economy (including, for example, interest rate derivatives) would have no effect on the equilibrium shown, as long as the unborn were still unable to participate in trading before date 1.[10]

Relative wealth concerns, however, mean that for low levels of bias, other young agents are induced to herd and trade with the bias rather than against it, causing m to increase with σ initially. When relative wealth concerns are sufficiently strong, as in this example, a speculative equilibrium is possible.

2.2 Stability

Under the conditions of Proposition 1, both the benchmark and a speculative equilibrium exist. However, we argue that the more natural outcome is in fact the equilibrium that exhibits price distortions. One refinement criteria that has been used in the literature is that of tâtonnement *stability*.[11] The definition of stability relies on an iterative procedure in which agents update their strategies on the basis of the current outcome. A stable solution has the property that if the economy is nearby, agents' best

[9] Note that in the speculative equilibrium illustrated, the unborn would like to hedge against high interest rates (they need to borrow at date 1), and so would sell the risky security if they could trade at date 0.

[10] In other words, because there are only two endogenous states, with two securities markets are already effectively complete, and adding new securities will not change the equilibrium.

[11] The first to study tâtonnement stability was probably Leon Walras; a more recent reference can be found in Mas-Colell, Whinston, and Green (1995, see section 17.H).

responses will move the economy closer to the equilibrium (rather than farther away).[12] If we imagine a process in which young agents gradually update their portfolios to be an optimal response to the choices of other young agents, we are led to the following notion of stability:

Definition 1. *An equilibrium σ is locally stable if for every $\hat{\sigma}$ in a neighborhood of σ and $k > 0$, the process $\sigma(t)$ defined by $\sigma(0) = \hat{\sigma}$ and $\sigma'(t) = k(m(\sigma(t)) - \sigma(t))$ converges to σ. An equilibrium is globally unstable if for any $k > 0$ and $\sigma(0) \neq \sigma$, the process $\sigma(t)$ does not converge to σ.*

In a one-dimensional context, stability is equivalent to the reaction function crossing the 45-degree line from above. As we can see in Figure 1, the speculative equilibrium is stable: when σ is below the equilibrium, young agents increase their holdings of the risk security, and when σ is above the equilibrium, they reduce their holdings. The benchmark equilibrium, however, is globally unstable. This holds generally, and follows from the fact that $m'(0) > 1$ and $m(1) < 1$, as we show in the proof of Proposition 1. Hence we conclude that:

Proposition 2. *Under the conditions of Proposition 1, the benchmark equilibrium in which $\sigma = 0$ is globally unstable and the speculative equilibrium is locally stable.*

2.3 Aggregate risk

Thus far, we have assumed all endowments are riskless. This setting is simple and demonstrates the existence of a bubble; still the benchmark equilibrium also exists (though it is unstable). Here we show that with a small amount of aggregate risk, an equilibrium in which young agents take excessive risk always exists and is often unique.

Suppose that the date 1 endowment of the middle-aged agents is $e_{M1}(1 + \sigma_0)$ in the event of rain, and $e_{M1}(1 - \sigma_0)$ in the event of no rain, for some $\sigma_0 > 0$. Similarly, young agents have endowment $e_{Y1}(1 + \sigma_0)$ and $e_{Y1}(1 - \sigma_0)$ in the events of rain and no rain, respectively. Thus, $\sigma_0 > 0$ represents the aggregate risk faced by middle-aged and young agents.

We compute the optimal reaction function m for the young agents (and similarly m_o for the middle-aged agents) as before, adjusting pricing and budget Equations (4) and (5) to account for the aggregate endowment risk

[12] One can view stability as corresponding to the iteration of "fictitious play" in the minds of the agents. A stable equilibrium is a limiting outcome of such a process.

at date 1:

$$\frac{p_0}{2-p_0} = \left(\frac{(e_{Y1}+e_{M1})(1+\sigma_0)-c(1+\sigma)}{(e_{Y1}+e_{M1})(1-\sigma_0)-c(1-\sigma)}\right)^{-\gamma},$$
$$e_{Y1}(1+\sigma_0(p_0-1)) = c(1+\sigma(p_0-1))$$

In this case, we interpret σ as the total risk of the young agent's portfolio; thus, given their endowment risk of σ_0, the difference $\sigma - \sigma_0$ is the risk induced by speculative trading. A speculative equilibrium is therefore one in which $\sigma > \sigma_0$.

Note that both types of agents hold an equal share of the aggregate date 1 endowment risk in proportion to their wealth. With CRRA utility, this allocation is Pareto optimal, and there would be no trade in equilibrium if there were no relative wealth concerns (that is, if the economy ended on date 1, or if the unborn could also trade at date 0). As Proposition 3 shows, however, with relative wealth concerns the young agents will hold excessive risk:

Proposition 3. *With aggregate risk $\sigma_0 > 0$, $m_0(\sigma_0) = \sigma_0$ and there would be no speculative trade absent relative to wealth concerns. However, if $\gamma > 1$, $m(\sigma_0) > \sigma_0$, and there exists a locally stable equilibrium with $m(\sigma) = \sigma > \sigma_0$ in which the young agents hold excessive risk.*

Proposition 3 shows that in the presence of aggregate risk, we no longer need condition (6) for relative wealth concerns to bias the equilibrium outcome (i.e., we no longer need restrictions on the relative wealth of the different cohorts). It is enough that $\gamma > 1$.

In equilibrium, the young agents will drive up the price of the risky security, driving down its risk premium. But this will not automatically lead to a bubble under our definition. With aggregate risk the risky security now has a positive risk premium in the benchmark equilibrium, and the young must trade sufficiently to drive down its risk premium enough so that the risk premium becomes negative. A negative risk premium will induce the middle-aged agents to sell the risky asset beyond the point at which they have completely hedged their endowment risk; that is, $m_0(\sigma) < 0$.

Figure 2 illustrates an example in which a bubble occurs. In this example $\gamma = 4$, all expected endowments are equal to 1, and $\sigma_0 = 10\%$. Note that this example does not satisfy condition (6), and that absent relative wealth concerns, $m_o(\sigma_0) = \sigma_0 = 10\%$ is the unique equilibrium. In our model, however, the unique equilibrium occurs with $\sigma = m(\sigma) = 30.8\%$, as shown in the figure, and $c = 0.94$. The position of the young agents is large enough so that the middle-aged agents hold a portfolio that implies that their wealth at time one is negatively correlated with the aggregate market risk, with bias $m_o(\sigma) = -8.4\%$. Thus the equilibrium risk premium of the risky security is negative with a relative price of $p_0 = 1.33$.

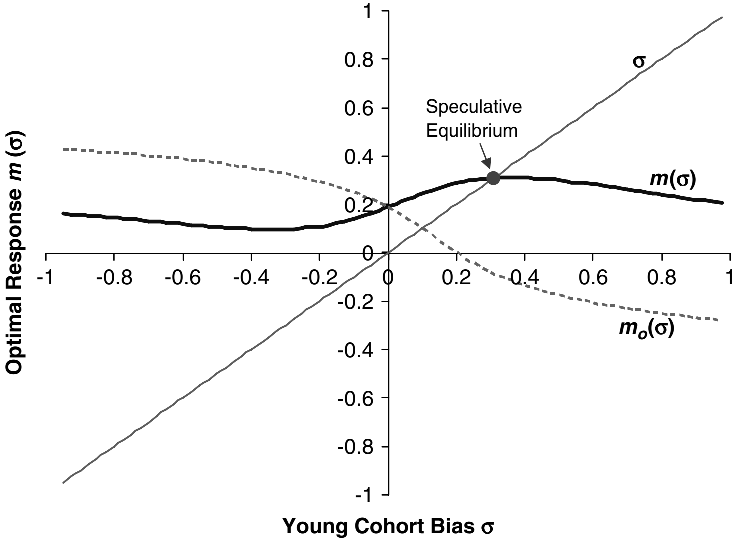

Figure 2
Equilibrium with aggregate risk
The economy has aggregate risk $\sigma_0 = 10\%$, which is the Pareto optimal bias ($m_o(\sigma_0) = \sigma_0$). Young agents herd and hold risk of over 30% in the unique equilibrium, however, leading to a negative risk premium.

2.4 Creating bubbles

What factors might cause bubbles to emerge in this economy? On the basis of our results thus far, two factors may lead to the emergence of a bubble:

2.4.1 A demographic change. Condition (6) implies that the likelihood of a bubble increases with the wealth of the middle-aged agents relative to the young agents, as the young agents have a smaller impact on current security prices. The likelihood also increases with the wealth of the young agents relative to the unborn agents, as young agents have a larger impact on next period's investment opportunities. Thus, for example, a projected demographic decline (such as the one discussed recently in the context of the crisis in the social security system) may make the benchmark equilibrium unstable and lead to a bubble.

It is also the case (as we will show in the next section) that an increase in the wealth of the young cohort can have a nonmonotonic effect so that a bubble exists only if the wealth of the young agents is neither too high nor too low. As a result, bubbles might emerge or collapse as the relative wealth of the young agents fluctuates.

2.4.2 An increase in aggregate risk. As we have seen, the presence of aggregate risk can induce a bubble in equilibrium even when the benchmark equilibrium is stable absent aggregate risk. The intuition for this outcome is that with aggregate risk, young agents are forced to hold some of the risky security. But once they hold some of the risk, the herding effect of their relative wealth concerns induces them to hold much more of it, leading to a bubble. This effect increases with the amount of aggregate risk in the economy. Thus, an increase in aggregate risk could lead to the emergence of a bubble.

There are several other mechanisms that may also lead some young agents to hold risky portfolios and induce a herding response by the cohort:

2.4.3 Portfolio constraints. If some agents are constrained to hold undiversified portfolios (for example, as part of their labor income due to moral hazard), then if indirect utility exhibits a herding incentive, these agents will "pull" other community members towards undiversified portfolios.

2.4.4 Differences of opinion. Agents may have different priors. For example, some young agents might be optimistic about the fundamentals of the risky security. Once these agents buy the security, relative wealth concerns of other young agents will "pull" them to trade in the same direction as the optimistic investors.[13] Thus, rational young agents amplify the bias, instead of absorbing it, leading to emergence of a bubble equilibrium. Note that the instability of the benchmark equilibrium and the stability of the bubble equilibrium implies that even a very small "optimism shock" on part of a subset of the young cohort can shift the economy away from the benchmark equilibrium to the speculative equilibrium.

2.5 Multiple speculative trading rounds

The relative wealth concerns of young agents can create distortions in the equilibrium prices of risky assets with just one round of speculative trade. We now show that with multiple rounds of speculative trade, bubble-like price dynamics can emerge.

In this equilibrium, as long as price distortions persist, they become more severe with time. The amplification along the bubble's path is due to the link between the returns of the risky assets and the wealth of the young cohort. As mentioned above, the young agents' portfolios are tilted towards the risky security. As a result, if it rains their wealth increases.

[13] In a static one period model Miller (1977) argues that in the presence of short-sale constraints pessimists are sidelined from investing, so the optimist is the only marginal pricer leading to higher valuations. Harrison and Kreps (1978) were among the first to demonstrate how differences in beliefs across investors can lead to a speculative premium given an investor buys under the belief he can resell next period to someone with higher beliefs in the future. In their setup all investors face a short-sale constraint and the market price is higher than all investors' marginal valuations for the stock.

However, as they become wealthier, their impact on asset prices increases, leading to a more pronounced price distortion in the following period.

Even more striking is the fact that the gains to the young agents if the bubble persists are much smaller than the losses they suffer when it bursts. To an outside observer it may seem that, in equilibrium, agents are risk loving even though they are in fact risk averse.

To introduce multiple speculative trading opportunities, we partition the time interval between date 0 and date 1 into subperiods $(0, \Delta, 2\Delta, \ldots N\Delta = 1)$. We refer to each subperiod of length Δ as a "day" and allow for speculative trade on each day j (time $t = j\Delta$) for $j = 0, \ldots, N-1$. As before, at date 1 the young and unborn agents engage in nonspeculative trade by exchanging consumption at date 1 for consumption at date 2. The revised endowment structure and timeline are illustrated in Table 3.

On each trading day $j = 0, \ldots, N-1$, young and middle-aged agents can trade two zero-net-supply securities. These securities have the following payoffs on each trading day:

- The risk-free bond pays a dividend of one unit of date 1 consumption each day.
- The risky security pays a dividend of two units of date 1 consumption each day that it rains. It pays no dividend each day that it does not rain. We assume the probability of rain on any day is 50%, independent of the past.

Note that the dividends of the securities are all for date 1 consumption—there is no consumption prior to date 1, only opportunities for additional speculative trade between the two types of agents.

The solution method is similar but more involved than the one used in Section 1.1. Here we rely on a numerical solution using dynamic programming. Working backward from date 1, and each period we compute the optimal portfolio choice of a young agent when the aggregate bias in portfolios of the young cohort on day j is σ. All other agents are assumed to be maximizing their utilities. A fixed point of the optimal response function corresponds to an equilibrium on day j.

Specifically, let $v_j^i(w, W_j)$ be the indirect utility function for a type $i = Y, M$ agent with date 1 wealth of w as of day j, when the current aggregate date 1 wealth of the young agents is W_j. For example, on day N (=

Table 3
Endowments of middle-aged, young and unborn agents with multiple periods of speculative trade

Date	0	Δ	...	$(N-1)\Delta$	1	2
Middle-aged	0	0	0	0	e_{M1}	—
Young	0	0	0	0	e_{Y1}	0
Unborn	—	—	—	—	0	e_{U2}

date 1), $v_N^Y(w; W_j) = v(w; W_j)$ defined in (3), and $v_N^M(w; W_j) = \frac{1}{1-\gamma} w^{1-\gamma}$. We compute the optimal response function $m_{j-1}(\sigma, W_{j-1})$ as follows. Given $W_j = c_j(1 \pm \sigma)$, the budget constraint (given W_{j-1}) and optimality condition for type M traders (implied by v_j^M) determine the expected wealth c_j and relative price p_j. Taking these as given, a young agent then chooses an optimal portfolio (which maximizes v_j^Y). An equilibrium in period $j-1$ is a fixed point $m_{j-1}(\sigma_{j-1}, W_{j-1}) = \sigma_{j-1}$. Given the equilibrium, we then determine the indirect utility functions v_{j-1}^i and work backwards.

In Figure 3 we present two graphs that summarize the solution we obtain for an example with three days of speculative trade.[14] The top graph represents the portfolio bias of the young agents as a function of their relative wealth, by displaying their wealth on the next day in the event of rain and no rain, respectively. The bottom graph displays the price of the risky asset (relative to the riskless asset), again as a function of the young agents' relative wealth. For an initial wealth of 0.5, the top graph shows that at the initial date the young agents choose a portfolio such that their wealth on the next day (time $t = \Delta$) is 0.55 if it rains and 0.16 if it does not. The reason for this disparity of gain versus loss is that for a relative wealth of 0.5 the risky security is quite expensive ($p_0 = 1.73$), as shown in the bottom graph of the figure.

The two graphs in Figure 3 also show that price distortions do not occur if either the young generation or the middle-aged generation is sufficiently wealthy: there are no distortions if either the aggregate relative wealth of the young generation is below 0.27 or above 0.82. Specifically, the bottom graph shows that in these regions the price of the risky is $p_j = 1$, and the top graph shows that in these regions young investors wealth on the next day remains the same and is not impacted by whether it rains or not. When the relative wealth of the young agents is too high, the impact of their trades on the price of the risky security is too great to support a speculative equilibrium. When the relative wealth of the young agents is too low, the impact of their wealth on the equilibrium interest rate R_1 at date 1 is too small to support a speculative equilibrium.

The price dynamics in the model can be inferred from Figure 3. Assuming that at the initial date the young agents' wealth is 0.5, the top graph shows that in this first day a young agent chooses a portfolio so that his wealth on the second day becomes 0.55 if it rains on the second day and 0.16 if it does not. The bottom graph shows that in this initial day a young cohort wealth of 0.5 implies a risky security price of 1.73. If, indeed, it rains on the second day so that the young agents' wealth goes up to 0.55, then on the second day a young agent will choose a portfolio leading to a wealth of 0.59 (0.16) on the third day if it rains (does not rain) on the third day.

[14] In the example, $\gamma = 4, e_{M1} = e_{Y1} = 0.5$ (without risk), and $e_{U2} = 0.3$. The equilibrium we derive is non-stationary; hence the graphs are slightly different each period.

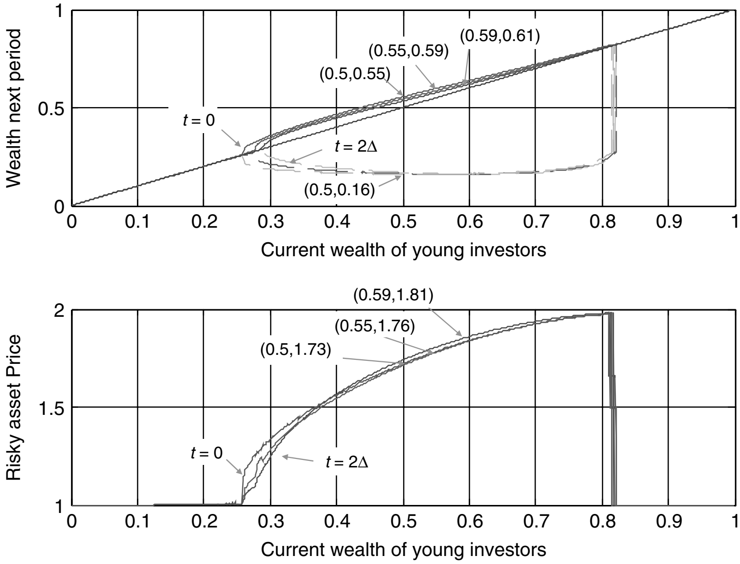

Figure 3
Evolution of wealth (top) and relative price (bottom) as a function of Young Cohort Relative Wealth
In the top figure, the upper and lower curves show the next period wealth of the young agents in the event of rain and no rain, respectively. A speculative equilibrium exists only if relative wealth is within the range (0.26, 0.82). The bottom graph shows the price of the risky asset relative to the riskless asset as numeraire. Each panel depicts three overlaid graphs corresponding to dates $t = 0$, $t = \Delta$, and $t = 2\Delta$.

In addition, the price distortion is exacerbated and the price of the risky security on the second day is 1.76. If it does not rain on the second day, then the young cohorts' wealth goes down to 0.16, and as can be inferred from the top graph it stays at 0.16 from this point until date 1 (i.e. for the next two days). Thus, the bubble has collapsed, the price of the risky security is 1 thereafter, and the young agent cohort never recovers. This is a consequence of the fact that the young cohort's aggregate relative wealth needs to exceed 0.27 for a speculative equilibrium to exist. As mentioned above, in the event it rained both on the first and second day, the wealth of the young cohort on the third day will be 0.59. This implies an even higher relative price for the risky security of 1.81 and a wealth of 0.61 (if it rains) or 0.16 (no rain) on the fourth day (date 1).

The above dynamics of the three period example are illustrated in Figure 4. In each node we report two numbers. The top one is the wealth of the young cohort, while the bottom one is the price of the risky asset p_j. (On date 1 there is no further speculative trade and so only the wealth of the young cohort is shown.)

Overall, the behavior that is illustrated in the figure is quite striking. The young generation repeatedly invests in the risky asset that pays when it rains the next day. While their wealth does not increase much, the bubble

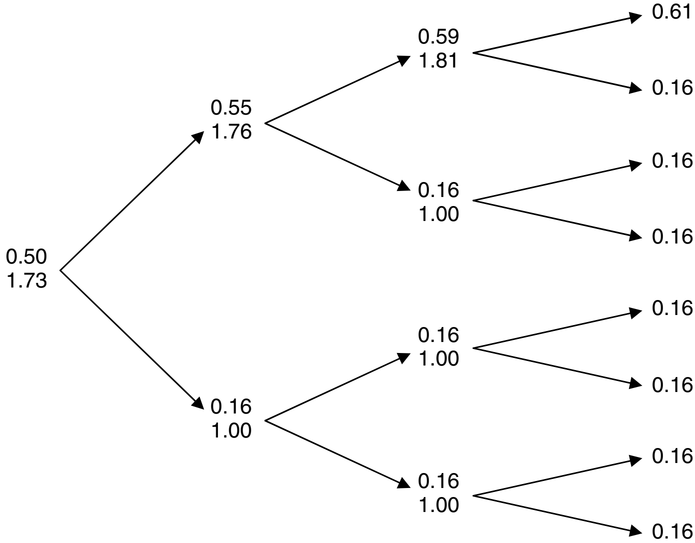

Figure 4
Dynamics of Young Cohort Relative Wealth (top) and the price of the risky asset relative to the riskless asset as numeraire (bottom) with Three Rounds of Speculative Trading
Up/Down arrows correspond to Rain/No Rain. Note that the pricing "bubble" grows until a No Rain event occurs, at which point it bursts.

grows owing to the feedback effect of their growing wealth along the bubble's path. If it does not rain one day, which is likely to happen, the bubble bursts. In this case the young generation is left with less than one third of its original endowment. Ultimately, the bubble bursts in seven out of eight cases. In those seven cases the interest rate $r_1 = 10.6\%$ is high, making it easy to save for retirement and enabling the young to consume $0.139/0.16 = 86.9\%$ of their wealth at date 1. In the single case in which it rains three consecutive days, they increase their wealth by only 20%. In this case, the interest rate $r_1 = -10.7\%$ is low, making it difficult to save for retirement and leading the young agents to consume only $0.064/0.61 = 11.5\%$ of their wealth at date 1. Hence, while the equilibrium is fully rational, endogenous relative wealth concerns yield price and portfolio dynamics that are surprising given that all agents are risk averse.

3. Consumption/Savings Gluts in a Deterministic Model

In the prior model, relative wealth effects induce herding into the risky asset, creating a price bubble. In this section we demonstrate that these same relative wealth effects can lead to distortions of the consumption/savings decisions of households within a cohort, even in a completely deterministic model.

The intuition for our result can be understood as follows. If, for example, a cohort has a high savings rate, it will depress the equilibrium interest rate. But then low interest rates increase the amount of saving needed to fund retirement. This feedback can lead to a savings glut in which interest rates are below the level consistent with aggregate fundamentals, and individual consumption growth is below aggregate consumption growth. Similarly, an alternative equilibrium can emerge with excessive consumption and interest rates that exceed fundamentals. We demonstrate these possibilities first in a finite-horizon overlapping generations model, and then consider the steady state as the horizon lengthens.

In this deterministic setting, endogenous relative wealth effects will lead to multiple equilibria, and can generate interest rates that are very different from what would be expected based on aggregate consumption growth. The very simple and standard nature of our setting also demonstrates that we must be very careful when evaluating overlapping generations models when generations span more than two periods, even when the horizon is finite.

Consider an N-period deterministic model with the endowment structure shown in Table 4. Apart from the first and the last generation, each generation lives for three dates. We label the three periods in an agents' life cycle as young, middle-aged, and old, where his endowment profile is $(\alpha, 3 - 2\alpha, \alpha)$. The first and last generations live for only two periods and have an endowment of $(2 - \alpha, \alpha)$ and $(\alpha, 2 - \alpha)$, respectively. In each date, other than the first and last ones, three cohorts coexist; in the first and last dates, only two cohorts interact.

The only decisions agents make at each date is how much to consume and how much to save. Specifically, while in the previous section agents were allowed to trade both in a risk-free bond and a risky stock, here we restrict agents to trading only in a risk-free zero-net-supply bond.[15] As before, agents cannot participate in financial markets prior to being born.

By construction, at each date aggregate endowment per capita equals 1, as each cohort has an average endowment of 1 per period. With this endowment structure, the standard benchmark equilibrium has a gross interest rate of 1 each period, and all agents smooth their consumption profiles so that they consume 1 unit per period. Because agents have an incentive to smooth consumption, one might guess that this equilibrium is unique. However, we show that there exists another equilibrium in which the gross interest rate exceeds 1, as well as a symmetric equilibrium with an interest rate below 1.

Similar to the stochastic case, we solve the model by computing the reaction function of the young agents at each date. At each date, only two

[15] Because agents only trade a risk-free bond, there is no loss of generality in not allowing agents to trade between consumption dates.

Table 4
Endowment structure for the N-period consumption/savings model

Date:	0	1	2	3	...	N	Endowment per period
Gen 1	$2-\alpha$	α	—	—	...	—	1
Gen 2	α	$3-2\alpha$	α	—	...	—	1
Gen 3	—	α	$3-2\alpha$	α	...	—	1
...	...	...	...	...	...	—	1
Gen $N-1$	—	—	—	α	$3-2\alpha$	α	1
Gen N	—	—	—	—	α	$2-\alpha$	1
Endowment per capita	1	1	1	1	1	1	

generations interact (young and middle-aged). The young agents realize that their cohort will trade in the future with the now unborn agents. The interest rates that will prevail are determined by the wealth of this unborn cohort as well as by how wealthy their own cohort will be. Specifically, the interest rate that will prevail when currently young agents turn middle-aged is a function of the overall wealth of their cohort at that time. As a consequence, the consumption/investment decisions of these young agents are endogenously linked to the decisions of the rest of their cohort so that a herding incentive emerges and agents imitate the actions of their cohort. Given this link, similar to before, we can derive the equilibrium by considering the reaction of a single young agent to the aggregate actions of other young agents, where equilibrium is a fixed point in which an individual young agent chooses the same action as his cohort.

We solve this model numerically. Figure 5 displays the evolution of the gross interest rate in a finite-horizon economy with 9 periods, γ of 3.5, and two values of α (0.1 for the solid line and 0.3 for the dashed line). The interest rates reported are annualized.[16] Note that a constant interest rate of 0% is one equilibrium for this economy. Nonetheless, even though the endowment is risk free, the horizon is finite, and agents are risk averse and rational, equilibria also exist in which interest rates differ from 0% for all periods.

For each choice of α, there is one equilibrium in which the initial interest rate is higher than 0% and then rises to a steady-state level before falling at the end. Given the symmetry of the model, given any equilibrium with a positive interest rate, there exists a second "mirror-image" equilibrium with a negative interest rate.[17] Table 5 shows the equilibrium consumption for each generation in the high interest rate equilibrium with $\alpha = 0.3$,

[16] Given that each agent's life cycle has only 3 dates, one should think of the time span between each two dates as relatively long. In the examples presented we assume it is 25 years.

[17] In particular, we can run time "backwards" in the model, and reinterpret an interest rate factor of R_t in period t as an interest rate factor of R_t^{-1} in period $N-t+1$.

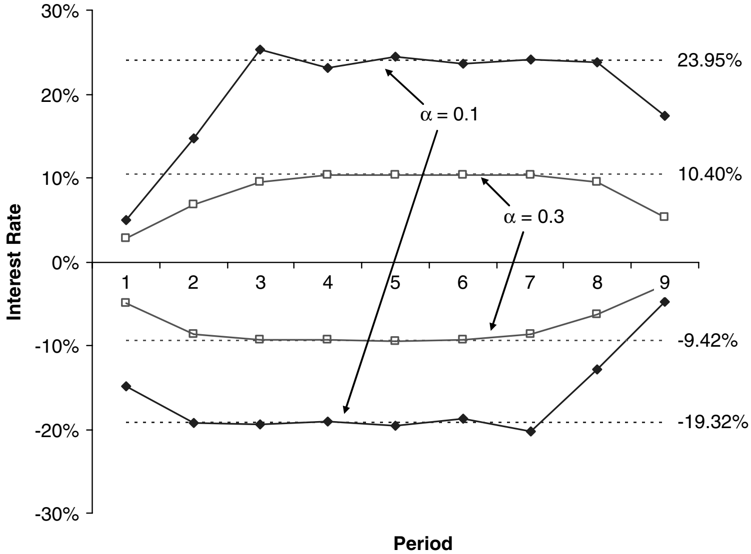

Figure 5
Equilibrium interest rates in a 9-Period Overlapping Generations Model with Endowment Parameter $\alpha = 0.1$ or 0.3 and $\gamma = 3.5$
In both cases, an interest rate of 0% in all periods is also an equilibrium. Dashed lines indicate the steady-state equilibrium interest rates.

Table 5
Consumption in the high interest rate equilibrium ($N = 9$, $\alpha = 0.3$, $\gamma = 3.5$)

Age	Gen 1	Gen 2	Gen 3	Gen 4	Gen 5	Gen 6	Gen 7	Gen 8	Gen 9	Gen 10
Y		0.85	0.56	0.45	0.42	0.42	0.42	0.42	0.44	0.55
M	1.15	1.04	0.90	0.85	0.85	0.85	0.85	0.85	0.83	0.79
O	1.40	1.65	1.72	1.73	1.73	1.73	1.71	1.62	1.21	—
ECR	1.25	1.04	0.77	0.64	0.61	0.61	0.61	0.61	0.62	0.63

together with the equivalent constant consumption rate (ECR) per period (i.e., the constant rate that provides an equivalent level of utility). Given the high interest rates, agents find it difficult to borrow when young, and put off much consumption until retirement. Early generations benefit as interest rates rise over their life. The pattern is reversed for the low interest rate equilibrium.

The existence of equilibrium with price distortions relies on γ being high enough and α being not too high. For example, one can show analytically that in an economy with only three dates when $\alpha = 0$ we need γ to be higher than 3 in order to obtain a herding equilibrium.

As we add more periods to this finite-horizon model, the equilibrium outcomes converge to a steady-state equilibrium of an infinite-horizon OLG model. This infinite-horizon model is a direct extension of the finite-horizon models, emphasizing the fact that the effect we focus on is different from standard findings in infinite overlapping generations models. Specifically, the behavior we characterize is due to herding and not fiat money.

In the infinite-horizon economy, each agent in the economy has the same endowment profile $(\alpha, 3-2\alpha, \alpha)$. In a stationary equilibrium, the gross interest rate is constant and is given by R. As a result, the consumption for each agent takes the form of $(c, cR^{1/\gamma}, cR^{2/\gamma})$. Agents live for three periods each so that in every period there are three generations that overlap. As a result, three is the total supply in each period and market clearing implies:

$$c(1 + R^{1/\gamma} + R^{2/\gamma}) = 3 \tag{7}$$

Next, the budget constraint for each agent implies

$$c + cR^{1/\gamma}/R + cR^{2/\gamma}/R^2 = \alpha + (3-2\alpha)/R + \alpha/R^2 \tag{8}$$

Combining (7) and (8), the equilibrium interest rate R can be characterized by:

$$3(1 + R^{1/\gamma-1} + R^{2/\gamma-2}) = (\alpha + (3-2\alpha)R^{-1} + \alpha R^{-2})(1 + R^{1/\gamma} + R^{2/\gamma}) \tag{9}$$

Using the above equations, it is not too difficult to show that:

Proposition 4. *In the steady state, $R = 1$ is an equilibrium, and if R is an equilibrium then so is R^{-1}. If $\gamma > \frac{2}{1-\alpha}$ then there exists an equilibrium in which $R>1$.*

The Proposition shows that for α close to zero we need γ to be only higher than 2 (compared with 3 in a three-period model). Indeed, one can show that adding more periods magnifies the herding effect. The proposition also shows that if the endowment is closer to the Pareto-efficient frontier (higher α), a higher risk aversion coefficient is needed for multiple equilibria to emerge.

The dashed lines in Figure 5 show the steady-state interest rates for $\gamma = 3.5$ and $\alpha = 0.1$, 0.3. In the steady state, agents are worse off in these equilibria than in the benchmark zero-interest-rate equilibrium. As the figure shows, the steady-state interest rates are closely approximated during the middle periods of the finite-horizon model.

4. Relative Utility and Keeping Up with the Joneses

In the previous sections we examined a model in which agents cared only about their own consumption. Despite this fact, some agents were sensitive to the wealth of others. This was due to the fact that wealth of others had an impact on prices, which in turn affected consumption. But our results are also relevant for models with exogenous specifications of relative consumption concerns based on "keeping up with the Joneses" (KUJ hereafter) type preferences. We derive a necessary condition for an exogenously specified preference regarding relative wealth to produce the herding effects needed to sustain a bubble. As we shall see, this condition essentially requires that the KUJ effect be sufficiently strong that it dominates the agent's risk aversion.

Consider an exchange economy in which the endowment is risk free. If all agents are identical, then even relative wealth effects will not have an equilibrium impact, as all agents will hold the same riskless consumption profile. For herding and price bubbles to emerge, at least two types of agents must be present in the economy. Hence, suppose there are two types of agents, the first type are agents who have standard CRRA utility functions $u(c) = \frac{1}{1-\gamma}c^{1-\gamma}$, the second type are agents who care explicitly about the consumption C of other agents of their own type as given by the function $u(c, C)$.

A key to obtaining price distortions is to have agents herd in their portfolio choice. Crowd-following behavior will potentially arise if agents' marginal utility from consumption (wealth) is higher when their community's average consumption (wealth) is high. Based on Abel (1990), and following Gali (1994) and Dupor and Liu (2003), we refer to this property as *keeping up with the Joneses*, defined as:

Keeping Up with the Joneses (KUJ): The agent's marginal utility is increasing in average community consumption, $u_{12}(c, C) > 0$.

With KUJ preferences, agents will choose to align their investment decisions more closely to their community members' decisions so as to be wealthy when others are wealthy. Thus, agents have an incentive to have a positive correlation between their portfolios and the portfolios of others. While KUJ preferences lead agents to herd and accentuate any biases that may exist regarding portfolio choice, the KUJ property by itself is in fact not sufficient to lead to the creation of self-sustaining biases and price bubbles. For example, consider a frictionless exchange economy, and the specification used in Gali (1994) in which

$$u(c, C) = f(c/C) = \frac{1}{1-\gamma}\left(\frac{c}{C}\right)^{1-\gamma} \tag{10}$$

In this case, when $\gamma > 1$ KUJ is satisfied. However, as Gali (1994) notes, this functional form leads to a unique outcome, which is identical to the outcome with a standard logarithmic utility function.[18] To see this, consider a simple case in which the aggregate endowment is risk free and there are only two equally likely states. Suppose the average consumption in the community is 100 units in one state and 150 in the

[18] One can easily generalize Gali's claim and show that it holds for any increasing concave function $f(c/C)$.

other; that is, there is herding into the second state. If an agent chooses this consumption profile, his utility is given simply by $f(1)$, and he avoids all relative wealth risk. By shifting a unit of consumption to the lower state, so that his consumption profile becomes (101,149), the agent gains more relative consumption in the low state than he gives up in the high state. Hence, individuals will choose to deviate from optimal risk sharing less than the rest of their community.[19] In equilibrium, this unravels a bubble equilibrium and implies that only optimal risk sharing can be sustained.

The bubble equilibria we present here have the striking feature that young agents and middle-aged agents choose wealth profiles that are negatively correlated. Specifically, wealth profiles are not monotonic functions of the aggregate endowment. Such an outcome can be an equilibrium only if marginal utility is nonmonotonic as we increase both individual and aggregate wealth over some range of equilibrium consumption. That is, there must be regions such that

$$\frac{\partial}{\partial c}u_1(c,c) = u_{11}(c,c) + u_{12}(c,c) > 0 \tag{11}$$

We refer to this property as a herding property of the preferences[20]:

Herding: The KUJ effect dominates the agent's risk aversion. Specifically, for some (c, C) where $C = c$, $u_{12}(c, C) > -u_{11}(c, C)$.

Using the above definition and discussion we conclude that:

Proposition 5. *An equilibrium in which agents' wealth is negatively correlated is possible only if preferences over wealth exhibit a herding property.*

In our model, the young agent's indirect utility over wealth can be written as

$$v(w, W_{Y1}) = \frac{1}{1-\gamma}w^{1-\gamma}\left[1+\left(\frac{W_{Y1}}{e_{U2}}\right)^{\gamma-1}\right]^{\gamma} \tag{12}$$

In this case, the herding property is equivalent to

$$\left(\frac{W_{Y1}}{e_{U2}}\right)^{\gamma-1} > \frac{1}{\gamma-2} \tag{13}$$

[19] This deviation is feasible if the price of consumption in each state is equal. But in fact, since the community has herded into the second state, it is likely that they have driven the price of consumption higher in that state, making the defection even more profitable.

[20] It is important to distinguish between what we refer to as a herding incentive and the herding behavior that is described in Banerjee (1992) and Bikhchandani, Hirshleifer, and Welch (1992). In our model there is no learning and all agents share the same information.

which requires $\gamma > 2$, and holds for $\gamma > 3$ and $W_{Y1} > e_{U2}$, which is implied by our earlier condition (6) for the existence of a herding equilibrium without aggregate risk. The two conditions coincide if the young cohort is small ($e_{Y1}/e_{M1} \to 0$); otherwise, condition (6) is stronger because the young agents' herding tendency must be strong enough to overcome their own impact on security prices. The weaker condition (13), however, is sufficient to guarantee an equilibrium with a negative risk premium in the presence of a small amount of aggregate risk.

Without the herding property, asset prices will still be affected because of the KUJ effect, which will lower risk premia. Bubble-like distortions, with negative premia, can only occur if some agents are constrained in their portfolio choice, as these constrained agents will tend to "pull" other agents to correlate with them. For example, Gomez, Priestley, and Zapatero (2006) introduce "workers" with a stochastic, nontraded endowment. These constrained agents are the source of the distortions in their model of exogenous relative wealth concerns.

5. Conclusion

In this article we have demonstrated that endogenous relative wealth concerns may play an important role in explaining the presence and dynamics of financial "bubbles". We have highlighted this fact by presenting a simple stylized *finite*-horizon fully rational overlapping generations model, in which agents care only about their own consumption.

When there are scarce goods whose price increases with the wealth of investors, agents' ability to consume will depend on their relative wealth. This externality induces a herding incentive: agents choose to imitate the portfolio choices of their cohort to avoid being poor when their cohort is wealthy. As a result, young investors may herd into the risky asset, driving up its price. The price distortion grows over time since as the young become wealthier, their impact on asset prices grows, further strengthening the herding incentive. Herding incentives are a key component of the mechanism that creates and sustains the bubble: highlighting the fact that a bubble is a social phenomenon.

The endogenous relative wealth concerns we identify will have an aggregate impact on equilibrium outcomes as long as there are some frictions that prevent agents endowed with the scarce goods from completely selling their endowment in advance. In our overlapping generations model these frictions are manifested by the inability to trade securities prior to birth, thereby leading to the emergence of relative wealth concerns for older generations of consumers. But other restrictions on participation would have the same effect. For example, one can describe a model in which moral hazard considerations prevent some agents from selling their future labor income. In this case the price of labor services will depend on community wealth, and lead to relative wealth effects in equilibrium.

An alternative, preference-related approach is to explicitly inject relative *consumption* considerations directly into the utility function. Specifically, we show that *keeping up with the Joneses* preferences alone are not sufficient to sustain herding in equilibrium, and derive a stronger necessary condition: the KUJ effect should dominate the agent's risk aversion (the sum of the cross-derivative and second derivative should be positive). We demonstrate that for both approaches there are two key requirements that are required for relative utility considerations to lead to bubble-like price dynamics in equilibrium. First, relative utility considerations must be sufficiently strong. Second, there should exist sufficient heterogeneity across agents' preferences so that agents can be separated into distinct *communities* on the basis of their preferences. In equilibrium, these communities trade with each other, breaking the link between aggregate consumption and asset prices. With identical agents, a representative agent exists and in equilibrium no price distortions will arise.

In reality, nonpecuniary preference-based relative wealth concerns may be important. Driving a BMW is likely to provide a different level of utility when neighbors are driving Honda Civics versus Jaguars. Such 'status' effects are an example in which relative utility emerges and cannot be captured as a pure pecuniary effect. These two relative utility channels need not be mutually exclusive and instead can complement one another. Combining the two approaches will strengthen the effects that we document and lead to a greater likelihood and severity of price bubbles, as might be expected.

Price distortions emerge in our model despite the fact that the horizon is finite, so that agents use backward induction in solving their respective portfolio choice problems. Financial "bubbles" exist even though agents have riskless endowments, and there is neither aggregate risk nor information asymmetry. In contrast to standard models in which the activity of "buy low/sell high" eliminates price distortions in equilibrium, if agents are sensitive to the wealth of others, trading against the crowd increases the risk of their relative wealth. As a result, rational traders may sustain prices that are too high even though they understand that these prices deviate substantially from fundamentals. In fact, given the price dynamics and the portfolio allocations produced by the model, an *outside observer* that does not account for relative wealth considerations might be tempted to conclude that some of the agents in the model are actually risk loving.

Our focus has been on financial bubbles. Empirically, there is a sense that at least some bubble-like episodes tend to occur in times when there is a potential of substantial technological innovation that has a significant level of uncertainty regarding its success. For example, the recent Internet bubble was associated with technology that was received with major enthusiasm by some and skepticism by others. The relationship between relative wealth concerns and technology bubbles is considered in DeMarzo, Kaniel, and Kremer (2007).

Appendix A:

Proof of Proposition 1

Normalizing $e_{M1} + e_{Y1} = 1$, market clearing implies that type M agents consume $1 - c(1+\sigma)$ in the event of rain, and $1 - c(1-\sigma)$ if it does not rain. Letting p be the price of the risk security, which pays 2 or 0, relative to the risk-free security (which pays 1 with certainty), the optimality condition for type M agents implies that security prices satisfy

$$\frac{p}{2-p} = \left(\frac{1-c(1-\sigma)}{1-c(1+\sigma)}\right)^{\gamma} \tag{A1}$$

The budget constraint of the young agents is given by

$$e_{Y1} = c(1+\sigma(p-1)) \tag{A2}$$

Using these two equations, we can solve for p and c, given σ. Let the optimal bias for a young agent be m. Given the young agent's indirect utility function (3), the young agent's optimality condition is

$$\frac{p}{2-p} = \left(\frac{(1-m)h(c(1+\sigma)/e_{U2})}{(1+m)h(c(1-\sigma)/e_{U2})}\right)^{\gamma} \tag{A3}$$

where $h(z) = \ 1 + z^{\gamma-1}$. The optimal response function $m(\sigma)$ is determined implicitly by the above three equations. The optimal response m_0 is determined in the same way, replacing (A3) with

$$\frac{p}{2-p} = \left(\frac{(1-m)}{(1+m)}\right)^{\gamma} \tag{A4}$$

Note that for $\sigma = 0, c = e_{Y1}$ and $p = 1$, so that $m(0) = 0$. Note also that for $\gamma > 1$, $m(1)<1$. Thus, existence of an equilibrium with $\sigma > 0$ and $p > 1$ is ensured if $m'(0) > 1$. Tedious algebra demonstrates that $m'(0) > 1$ is equivalent to (6).

Proof of Proposition 3

With aggregate risk, equilibrium conditions (A1) and (A2) become

$$\frac{p}{2-p} = \left(\frac{1-\sigma_0-c(1-\sigma)}{1+\sigma_0-c(1+\sigma)}\right)^{\gamma} \tag{A5}$$

$$e_{Y1}(1+\sigma_0)p + e_{Y1}(1-\sigma_0)(2-p) = c(1+\sigma)p + c(1-\sigma)(2-p) \tag{A6}$$

Note that from (A4), (A5) and (A6), $m_0(\sigma_0) = \sigma_0$. However, since h is increasing when $\gamma > 1$, we conclude that $m(\sigma_0) > \sigma_0$. Since $m(1)<1$ we conclude that the reaction function crosses the 45 degrees line from above at some $\sigma > \sigma_0$.

Proof of Proposition 4

The first two parts are immediate from the condition.

$$3(1 + R^{1/\gamma-1} + R^{2/\gamma-2}) = (\alpha + (3{-}2\alpha)R^{-1} + \alpha R^{-2})(1 + R^{1/\gamma} + R^{2/\gamma}) \tag{A7}$$

To show the third part, define

$$f[R] \equiv 3(1 + R^{1/\gamma-1} + R^{2/\gamma-2}) - (\alpha + (3-2\alpha)R^{-1} + \alpha R^{-2})(1 + R^{1/\gamma} + R^{2/\gamma}) \qquad \text{(A8)}$$

A direct calculation shows that at $f'[1] = 0$ and $f''[1] > 0$ iff $\gamma > \frac{2}{1-\alpha}$. Furthermore, when $\gamma > \frac{2}{1-\alpha}$ it is easy to see that for R large enough $f[R] < 0$. Combining these two observations yields the result.

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