



# Technological innovation and real investment booms and busts<sup>☆</sup>

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## Abstract

We investigate why new, high-risk technologies can attract excessive and often unprofitable investment. We develop an equilibrium model in which rational, risk-averse agents overinvest in a risky technology, possibly to the point that its expected return is negative. Overinvestment results from relative wealth concerns which arise endogenously from the imperfect tradability of future endowments. Competition over future consumption leads to an indirect utility for wealth with “keeping up with the Joneses” properties that can induce herding. Because overinvestment increases with the risk of the technology, our model can explain why new, risky technological innovations may promote investment bubbles.

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The optimism of stock market participants [in the mid-1990s] prompted entrepreneurs and firm managers to invest in capital assets... As growing business investment mirrored rising aggregate after-tax corporate profits from 1990 to 1997, this

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optimism appeared to be justified. However, business investment and aggregate after-tax corporate profits diverged from 1997 to 2000. Entrepreneurs and firm managers succumbed to the same “irrationality” regarding their decisions to invest in capital assets that stock market participants were suffering regarding their decisions to purchase equities.<sup>1</sup>

## 1. Introduction

The emergence of the internet in the mid-1990s led to a wave of technological innovation and a boom in both asset prices and real investment. The recession that followed is perhaps best described as an investment bust, for while consumption in the US continued to grow, corporate investment fell sharply following the failure of much of it.<sup>2</sup> In retrospect, much of this investment appears to have been excessive. While an ex-post view is often misleading, many argue that a significant part of the business investment in the late 1990s should not have been expected to yield a positive return under reasonable forecasts.

The telecommunication industry provides an illustrative example. Between 1996 and 2000, telecom companies raised \$1.25 trillion dollars in debt, a figure that reflects only a fraction of the capital raised. Much of this capital was spent on excessive investment in fiber optic communication lines, with no fewer than six companies creating parallel national communication networks and hundreds more laying local lines, so that by 2003, 39 million miles of fiber optic lines criss-crossed the United States. The result was a glut of capacity far in excess of expected near-term demand, and the utilization rate of telecom networks in 2003 fell to a disastrously low 2.5–3%. Not surprisingly, from December 2000 until the beginning of 2003 the price of these lines declined by more than 90%, and telecommunications companies accounted for 60% (by net worth) of US corporate bankruptcies.<sup>3</sup>

In many cases, as in the above example, bubble-like behavior coincides with periods and industries undergoing a concentration of technological innovation. Other examples include the Mississippi Scheme and the South Sea Bubble, in which the “technological innovation” was the granting of monopoly trading rights with the new world. Similarly, the stock market bubble of the 1920s was driven primarily by the new technology stocks of the time, namely the automobile, aircraft, motion picture, and radio industries. In the mid-1840s Britain experienced a “railway mania” with over twelve hundred railways under construction by 1845, at an estimated cost of over 560 million pounds and aggregate railway liabilities of nearly 600 million pounds. At the time, Britain’s national income was only 550 million pounds, and railway investments were starving the rest of the economy of capital. The end result of this uncontrolled expansion was a haphazard, redundant, and largely unprofitable railway network. By January 1850, the average receipts per mile of rail track had fallen by over a third, railway shares had declined from their peak by an average of over 85%, and the total value of all railway shares was less than half the capital invested.<sup>4</sup>

<sup>1</sup>“Economic repercussions of the stock market bubble,” Joint Economic Committee of the United States Congress (2003, p. 3).

<sup>2</sup>Corporate investment grew at a 14.5% annual growth rate in the first quarter of 2000 only to shrink to a –16% growth rate in the second quarter of 2001 (Ibid, p. 3).

<sup>3</sup>Ibid, p. 7.

<sup>4</sup>See Chancellor (1999).

In these examples, investors appeared to have over-invested in technology and ignored the risks involved. The goal of this paper is to provide a potential link between technological innovation and the emergence of investment bubbles, which we define as periods of excessive, and *predictably* unprofitable, investment. Most existing models of bubbles (such as Samuelson “fiat money” type models) focus on financial investments and do not contain any substantive link to the features of the underlying assets. They do not therefore provide an explanation for why technology stocks should be more susceptible to bubbles than, for example, utility stocks. In this paper we explore a model of relative wealth concerns based on DeMarzo, Kaniel, and Kremer (2004) and show that in such a model, risky technologies will attract excessive investment that can be largely unprofitable.

In our model, rational risk-averse agents have access to a risky production technology in addition to risk-free storage that yields one unit for every unit stored. To highlight some of our main results, consider a simple setting in which the risky technology yields  $k$  per unit invested with probability  $1/k$  and zero with probability  $1-1/k$ . We show in Section 4 that despite the fact that agents are risk averse and the risky technology has zero expected return,

- Agents/firms invest in the risky technology if it is sufficiently risky ( $k$  is sufficiently large).
- Investment in the risky technology is increasing in its risk (indexed by  $k$ ).

This result demonstrates that the introduction of a new risky technology can lead to investment behavior that may appear to be “irrational” for risk-averse agents. Because new technologies are likely to have skewed payoff distributions (i.e., a small chance of a very high return), they are more susceptible to these distortions.

The results in our model are based on the following key idea. First, we argue that when rational investors expect to compete with each other in the future over scarce resources, relative wealth concerns arise endogenously. Risk-averse investors fear being relatively poor when other investors do well, as they will be “priced out” of the market for future resources. As a result, investors’ marginal utility of income will increase with the wealth of others in their community. In other words, investors have an endogenous preference to “keep up with the Joneses.” We also show that this same effect can arise from firms’ concerns regarding the future price of labor services if the labor pool is a scarce resource.

The key departure from the standard general equilibrium model necessary for these relative wealth concerns to arise is that future scarce resources are not tradable (or available as collateral) in the market today without limitation or cost. This restriction is natural for many types of services, due to the moral hazard concerns that arise with human capital.<sup>5</sup> Also, many future resources may not yet exist as tangible assets, making them difficult to collateralize.

In our model, relative wealth concerns arise endogenously in equilibrium. One could alternatively assume that agents care directly about their wealth in relation to the wealth of others.<sup>6</sup> With a utility function similar to the indirect utility function we derive here, the same conclusions regarding investment would hold. In reality, we believe that both sources

<sup>5</sup>Additionally, in an overlapping generations framework, the future service providers may not be able to participate in markets today.

<sup>6</sup>Frank (1985) argues that such preferences are a natural outcome of individuals’ concern for social status.

of relative wealth concerns are likely to be operative (and complementary) in producing investment distortions. Our model has the advantage of avoiding an ad hoc specification of the utility function, relying instead on the sensitivity of the consumption good's price to the wealth distribution (see Aït-Sahalia, Parker, and Yogo (2004) for compelling evidence in favor of such effects). As a result, our approach yields a potentially richer set of testable implications. Ultimately, however, the focus of our paper is on the impact of relative wealth concerns on investment and the possibility of bubbles, rather than the origins of such preferences.

We focus on the consequences of endogenous relative wealth concerns on real investment.<sup>7</sup> We begin by considering a standard 2-period general equilibrium model with a risky and a riskless investment technology as a benchmark. We then show that with competition for future scarce goods, overinvestment in the risky technology occurs. Moreover, this overinvestment is increasing in the risk of the technology. We also demonstrate that overinvestment can occur to the point that a profitable technology's expected return to all investors is negative. This surprising result would likely be viewed by outside observers as evidence of a "bubble."

### 1.1. Related literature

There is an extensive literature on bubbles in financial markets. Rather than providing a full account of this literature we refer the reader to Brunnermeier (2001) for an excellent survey. The approach we follow here (and in a related paper) is somewhat unique. Our model is a fully rational general equilibrium model in which agents are characterized by a standard utility function. We do not rely on an infinite horizon, fiat money, or asymmetric information. The combination of these features is not shared by the prior literature.

Our approach models relative wealth considerations that arise endogenously. While several papers in economics and finance consider relative wealth effects, in almost all of these papers these effects are assumed exogenously as a part of preferences. That is, agents are explicitly assumed to care about other people's consumption. In the finance literature, Abel (1990) introduces relative considerations in his paper on "catching up with the Joneses." Abel (1990) and Gali (1994) consider this utility function as a potential resolution to the equity premia puzzle. Dupor and Liu (2003) analyze the potential impact of consumption externalities on over-consumption. This paper complements the above papers as it considers a different framework in which the main focus is on overinvestment in a risky technology.

Cole, Mailath, and Postelwaite (1992, 2001) consider a model in which non-market interactions give rise to relative wealth concerns. In their models, individuals compete for mates, and their success depends upon their relative wealth or "status." This non-market competition over mates has a similar effect as the market competition over scarce resources that occurs in our model. They consider the implications of relative wealth concerns for portfolio choice and investment rates, but do not focus on its implications for overinvestment in risky technologies.

In contemporaneous work, Pastor and Veronesi, 2006 argue that technological revolutions are characterized by large uncertainty about future growth, and that this

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<sup>7</sup>See DeMarzo, Kaniel, and Kremer (2006) for an exploration of the dynamics of security price distortions that can arise with relative wealth concerns.

uncertainty can generate high prices and excessive volatility initially, with prices falling as the technology is adopted and its risk becomes systematic. In contrast, in our model it is the systematic nature of the risk that can lead to an investment bubble.

Our paper is most closely related to DeMarzo, Kaniel, and Kremer (2004, 2006). Both papers examine exchange economies in which there are no production decisions and hence there is no role for technology. The first paper shows that when investors care about their consumption relative to their local community, there may be a community effect whereby investors under-diversify and over-invest in local firms.<sup>8</sup> DeMarzo, Kaniel, and Kremer (2006) complement the results of this paper by showing in a dynamic setting how endogenous relative wealth concerns can create bubble-like deviations in the prices of financial assets. As with much of the existing literature on bubbles, neither paper considers why bubbles arise in some environments and not in others. Specifically, they do not explain why the appearance of bubbles is often associated with the appearance of new technologies.

In this paper, we try to address this issue by focusing on the distortions in real investment that result from relative wealth concerns in a production economy. We highlight the positive relation between the riskiness — and in particular, the positive skewness — of the returns to a new technology and the degree of overinvestment.

The paper is organized as follows. In Section 2, we describe a two-date production economy with two goods, two types of agents, and two investment technologies. Section 3 begins with a characterization of first-best investment. We then show that in equilibrium agents overinvest in the risky technology relative to this benchmark. In Section 4, we parameterize the model so that a closed-form solution can be obtained, and show that overinvestment increases with the riskiness of the technology. We introduce competition in Section 5, allowing the return to investment to decrease with the aggregate investment in the risky technology. We show that producers may compete away all surplus and invest until the expected return from the technology is negative. In Section 6 we relate our results to models with exogenously specified relative wealth concerns, and in Section 7 we extend the model to include publicly traded firms.

## 2. The model

We consider a two-date stochastic production economy with two agent types: producers and service providers. There is a continuum with mass one of each type of agent. Producers live for two dates, while service providers exist only at the second date. At the first date, ( $t = 1$ ), producers can invest in two production technologies, one riskless and one risky, as well as trade securities. Both technologies produce their output on the second date ( $t = 2$ ), and both produce equivalent goods. At date 2, given the producers' output, producers and service providers trade goods and services competitively in the spot market and consume. All agents act to maximize their utility over final consumption of goods and services.

Note that, for simplicity, this specification does not distinguish between ownership and control. In Section 7 we present an equivalent model in which producers are instead investors who invest in public firms. These firms choose an investment strategy to

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<sup>8</sup>See also a recent paper by Gomez, Priestley, and Zapatero (2004), who develop a similar model and find empirical support for it in an international asset pricing context.

maximize their market value. (Because service providers do not participate in financial markets at date 1, this objective is unanimously preferred by all shareholders.) Such a model leads to the same equilibrium outcome that we derive here, but allows for further empirical interpretations of the results.

An important assumption in our model is that service providers do not invest or trade securities at date 1. As we shall see this participation constraint is the underlying source of the relative wealth concerns that drive the equilibrium in our model. The constraint can be motivated by moral hazard concerns and liquidity constraints. (An alternative to this assumption would be to directly assume relative wealth concerns as part of agents' preferences. We discuss this possibility in Section 6.)

In our model agents care only about their personal consumption; relative considerations emerge endogenously. All agents maximize expected utility over terminal consumption given by a separable CRRA utility function. Agents derive utility from both goods and services according to

$$u(c_g, c_s) = \frac{1}{1-\gamma} \left( c_g^{1-\gamma} + c_s^{1-\gamma} \right),$$

where  $c_g$  and  $c_s$  denote the consumption of goods and services, respectively. Note that the parameter  $\gamma > 0$  specifies agents' relative risk aversion. (It also determines the elasticity of substitution between the goods. One could specify more general preferences to separate these and obtain similar results.)

Service providers are endowed with one unit of services at date 2. Producers are endowed with one unit of an investment good at date 1, which they invest in either of the technologies to produce goods in date 2. Given that there is a mass of one of each type, these also equal the aggregate endowments.

An investment of  $x$  in the risky technology produces  $(1 + \theta)x$  goods at date 2, where  $\theta \geq -1$  is a random variable. If the producer invests the remaining  $(1-x)$  in the riskless technology, this technology will produce  $(1-x)$  goods at date 2 with certainty. Thus, total output is given by

$$(1-x) + (1+\theta)x = 1 + x\theta \text{ for } x \in [0, 1].$$

We also assume markets are complete in the sense that producers can trade arbitrary securities in date 1. Because agents have homothetic preferences, all idiosyncratic production risk will be shared. Thus, we can interpret  $\theta$  as representing the aggregate risk of the risky technology. For simplicity we assume a simple binary distribution for  $\theta$ : With probability  $\pi$  the technology is successful and generates output  $1 + \theta = k > 1$ , and with probability  $1-\pi$  the technology fails and generates output  $1 + \theta = 0$ .

Given this specification of preferences, endowments, and technology, we define an equilibrium in the usual way: Agents make investment, consumption, and trading decisions to maximize their utility, and spot markets clear.

### 3. Analysis

In this section we first consider the benchmark case without trading frictions and characterize the first-best level of investment. We then evaluate the equilibrium for our model, and show that the inability of producers and service providers to trade in period 1 leads to overinvestment in the risky technology.

### 3.1. First-best investment

Before analyzing the equilibrium of the model we consider a benchmark in which relative wealth does not affect the equilibrium outcome. Suppose for now that those endowed with services are unconstrained in their participation in securities markets at date 1. Without trading constraints, the first welfare theorem implies that the resulting equilibrium is Pareto optimal.

Standard aggregation results imply that the economy is equivalent to one with a single representative agent. Because aggregate consumption of services is fixed, the optimal investment decision maximizes the utility from the consumption of goods<sup>9</sup>:

$$\max \frac{1}{1-\gamma} E[(1+x\theta)^{1-\gamma}] \quad \text{s.t. } x \in [0, 1].$$

Let  $x_b$  denote the optimal choice in the benchmark case. Assuming an interior solution  $x_b \in (0,1)$ , the first-order condition is given by

$$(1-\pi)(1-x_b)^{-\gamma} = \pi(k-1)(1+x_b(k-1))^{-\gamma}.$$

Incorporating the additional constraint  $x_b \in [0,1]$ , the solution is given by

$$x_b = \max \left\{ \frac{1-\alpha}{1+(k-1)\alpha}, 0 \right\}, \tag{1}$$

where  $\alpha = ((1-\pi)/\pi(k-1))^{1/\gamma}$ . Note the following properties of the first-best level of investment:

- Investment in the risky technology  $x_b$  is increasing in the probability that this technology is successful (i.e.,  $x_b$  is decreasing in  $\alpha$  while  $\alpha$  is decreasing in  $\pi$ .)
- If the risky technology has a nonpositive expected return ( $\pi k \leq 1$ ), there will be no investment in it ( $\alpha \geq 1$  and  $x_b = 0$ ).
- If the risky technology has positive expected return ( $\pi k > 1$ ), then holding the expected payoff  $\pi k$  fixed, the investment in the risky technology  $x_b$  is decreasing in its risk,  $k$ .

In the following we will contrast these last two findings with our model. Note that the second result does not depend on the specific utility specification. Any risk-averse agent would not adopt such technology, as he would not be compensated for the assumed risk. We shall see that with relative wealth concerns, the risky technology may be adopted even if its expected return is negative and it reduces welfare. We will also show that investment in the risky technology increases with its risk,  $k$ .

### 3.2. Relative wealth effects and overinvestment

In our model, service providers do not participate in the market at date 1. As a result, we cannot analyze the equilibrium at date 1 from the perspective of a single representative agent. Instead, we solve the model by first analyzing the spot market equilibrium at date 2. With separable CRRA preferences, if the aggregate supply of goods in the second period is

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<sup>9</sup>Note that the optimal investment problem is equivalent to that for an economy without services or service providers.

given by  $Y$ , then the price of labor services (in terms of goods) increases with  $Y$  and is given by

$$p = Y^\gamma.$$

Given his budget constraint, a producer with a date-2 endowment of  $y$  goods chooses to consume  $y/(1 + p^{1-1/\gamma}) = y/(1 + Y^{\gamma-1})$  goods and  $y/(Y + Y^\gamma)$  units of labor services. This consumption profile implies an indirect utility of

$$V(y, Y) = \frac{1}{1 - \gamma} y^{1-\gamma} h(Y)^\gamma, \quad (2)$$

where  $h(Y) = 1 + Y^{\gamma-1}$ .

Let  $X$  be the aggregate level of investment in the risky technology. Then the aggregate supply of consumption goods in period 2 is  $Y = 1 + X\theta$ . If producers are unable to trade the risk of their production with service providers in date 1, then the date-2 income of an individual producer who invests  $x$  is  $y = 1 + x\theta$ . Each producer will therefore choose investment  $x$  to maximize her (indirect) utility:

$$\begin{aligned} & \max_{x \in [0,1]} E[V(1 + x\theta, 1 + X\theta)] \\ & = \max_{x \in [0,1]} \frac{1}{1 - \gamma} [(1 - \pi)(1 - x)^{1-\gamma} h(1 - X)^\gamma + \pi(1 + x(k - 1))^{1-\gamma} h(1 + X(k - 1))^\gamma]. \end{aligned}$$

The first-order condition assuming an interior solution is given by

$$(1 - \pi)(1 - x)^{-\gamma} h(1 - X)^\gamma = \pi(k - 1)(1 + x(k - 1))^{-\gamma} h(1 + X(k - 1))^\gamma.$$

In equilibrium the action of an individual producer matches the aggregate action of the producer population, that is  $x = X$ . The following result establishes that if agents are sufficiently risk averse (i.e., more risk averse than log utility), then relative wealth effects lead to overinvestment in the risky technology (relative to the first-best level,  $x_b$ ) when investment occurs.

**Proposition 1.** *Suppose  $\gamma > 1$ . Then if agents invest in the risky technology ( $x > 0$ ), investment exceeds the first-best level, that is,  $x > x_b$ . Otherwise,  $x = x_b = 0$ .*

**Proof.** The first-order condition for the agent's maximization problem can be written:

$$\frac{1 - x}{1 + x(k - 1)} \frac{h(1 + X(k - 1))}{h(1 - X)} = \alpha. \quad (3)$$

where  $\alpha$  is defined as in (1). An equilibrium with  $x > 0$  solves (3) with  $X = x$ . Compare this condition to the first-order condition in the benchmark case:

$$\frac{1 - x_b}{1 + x_b(k - 1)} = \alpha.$$

The proof follows from the fact that  $x > 0$  implies  $1 + x(k - 1) > (1 - x)$ , which, since  $h$  is increasing (given  $\gamma > 1$ ), implies that  $h(1 + x(k - 1))/h(1 - x) > 1$ .  $\square$

Proposition 1 says that as long as the risky technology is sufficiently attractive that some investment in it occurs (which is guaranteed, for example, if the technology has a positive expected return), then producers invest beyond the first-best level. The result follows from the fact that for  $\gamma > 1$ , the function  $h(Y)$ , and therefore the marginal utility of income,

$\partial V(y, Y)/\partial y$ , is increasing in aggregate output  $Y$ . Agents value income more when others succeed and output is high because competition raises the price  $p$  of services in the economy. The fear of being “left behind” and relatively poor when others succeed leads agents to correlate their investment in the risky technology. Though the investment is risky, investing when others do reduces the risk of their *relative* wealth, and producers invest more in the risky technology than they otherwise would. More abstractly, when  $\gamma > 1$  and service providers do not trade in period 1, investments by different producers are complements (the function  $V$  is supermodular), which increases the equilibrium level of investment.

Overinvestment in our model depends upon the trading frictions in period 1. If producers and service providers could trade a complete set of securities at date 1, the equilibrium outlined in Section 3.1 is the unique outcome. The intuition for the role of trading frictions is as follows. Because service providers’ income increases with the price  $p$  of services in period 2, which increases with  $Y$ , service providers would like to hedge their income risk and sell securities that are positively correlated with the output of the risky technology. Producers would then satisfy their desire to correlate their income with  $Y$  by buying these securities rather than investing more in the risky technology. Overinvestment therefore results as long as there exist frictions that make it costly for service providers to short-sell these securities to producers at date 1. (We restrict such trade altogether for simplicity.)

#### 4. Increasing risk and overinvestment

A key result of our paper is that increasing the risk of the technology may in fact induce more investment in this technology, yielding a larger deviation from the first-best outcome. This result arises because as risk increases, so does the concern of being “left behind” when others succeed. We demonstrate this result in several settings below.

In this section we simplify our model by considering the special case in which  $\pi = 1/k$ . That is, the project has zero expected return, with risk that is increasing in  $k$ . This case has two advantages. First, for some parameters we can find a simple closed-form solution. Second, because we would not expect risk-averse agents to invest in a mean preserving spread, we can clearly label such investment a “bubble.”

##### 4.1. A closed-form solution

Our model has a simple solution in the case  $\gamma = 3$ . Then,  $h(z)/z = z + 1/z$ , and because the project has zero expected return,  $\alpha = 1$ . As a result, condition (3) reduces (after some algebra) to

$$\frac{1}{1-x} = 1 + x(k-1).$$

This equation has two solutions:  $x = 0$  and  $x = (k-2)/(k-1)$ . Because negative investment is not feasible, if  $k \leq 2$  the unique equilibrium is  $x = 0$ . When  $k > 2$ , both solutions are feasible. Thus, we have the following characterization in this setting:

- When the technology is not sufficiently risky ( $k \leq 2$ ), the unique solution is  $x = 0$ . That is, the risky technology is not adopted if its risk is below this threshold.

- When the technology is sufficiently risky ( $k > 2$ ), there is an additional solution in which  $x = (k-2)/(k-1)$ . Thus, the amount invested in the risky technology is increasing in its risk. In the limit in which the technology becomes extremely risky, producers invest their entire endowment in this technology.

Clearly any investment in the risky technology lowers the utility of producers. We argue that service providers also suffer from the volatility introduced by this investment:

**Proposition 2.** *When  $\gamma > 2$  and the technology has zero expected return, the equilibrium with positive investment in the risky technology is Pareto dominated by the equilibrium in which only the riskless technology is utilized.*

#### *Equilibrium selection*

The analysis above shows the existence of an equilibrium level of investment that increases with the risk of the technology. However, it is not a unique equilibrium. One could argue that the efficient outcome  $x = 0$  will always be selected, and the equilibrium with investment can be safely ignored.

One refinement criterion that has been used to select equilibria in the literature is that of *stability*. The definition of stability relies on an iterative procedure in which agents react to the last period's outcome. A stable solution has the property that if the economy is nearby, agents' best responses will move the economy closer to the equilibrium (rather than farther away).<sup>10</sup> Define  $m(X)$  as the producers' optimal response to an aggregate level of investment in the risky technology,  $X$ . From the first-order condition (3), the optimal response or reaction function can be written as

$$x = m(X) = \frac{h(1 + X(k - 1)) - h(1 - X)\alpha}{h(1 + X(k - 1)) + (k - 1)h(1 - X)\alpha}. \quad (4)$$

Using this notation we define a stable equilibrium as follows:

**Definition.** An equilibrium  $x$  is *locally stable* if for every  $x'$  in a neighborhood of  $x$ , the sequence  $\{x_n\}_{n=0}^{\infty}$  defined by  $x_0 = x'$  and  $x_{i+1} = m(x_i)$  converges to  $x$ . An equilibrium  $x$  is *globally unstable* if any sequence  $\{x_n\}_{n=0}^{\infty}$  for which  $x_0 \neq x$  and  $x_{i+1} = m(x_i)$  does not converge to  $x$ .

An equilibrium  $x$  corresponds to a fixed point  $x = m(x)$ , whereas stability of an equilibrium  $x$  requires that  $|m(x + \varepsilon) - x| < \varepsilon$  for all  $\varepsilon$  close to zero. To check for stability of the solution  $x = 0$ , given  $\alpha = 1$ , we have that

$$m'(0) = (\gamma - 1)/2 \text{ and } m''(0) = (k - 2)(\gamma - 1)(\gamma - 2)/2.$$

When  $\gamma = 3$ , this implies that  $m'(0) = 1$  and  $m''(0) = k - 2$ . Thus, for  $k > 2$ ,  $m''(0) > 0$  and  $m(\varepsilon) > \varepsilon$  for  $\varepsilon$  close to zero. Therefore,  $x = 0$  is unstable. Similar analysis can be used to show that  $0 < m'((k-2)/(k-1)) < 1$ , so that  $x = (k-2)/(k-1)$  is stable. Thus, we have the following result:

**Proposition 3.** *When  $\gamma = 3$  and the technology has zero expected return, if  $k > 2$  the unique stable equilibrium is  $x = (k-2)/(k-1)$ . The equilibrium in which  $x = 0$  is globally unstable.*

<sup>10</sup>One can view stability as corresponding to the iteration of "fictitious play" in the minds of the agents. A stable equilibrium is a limiting outcome of such a process.

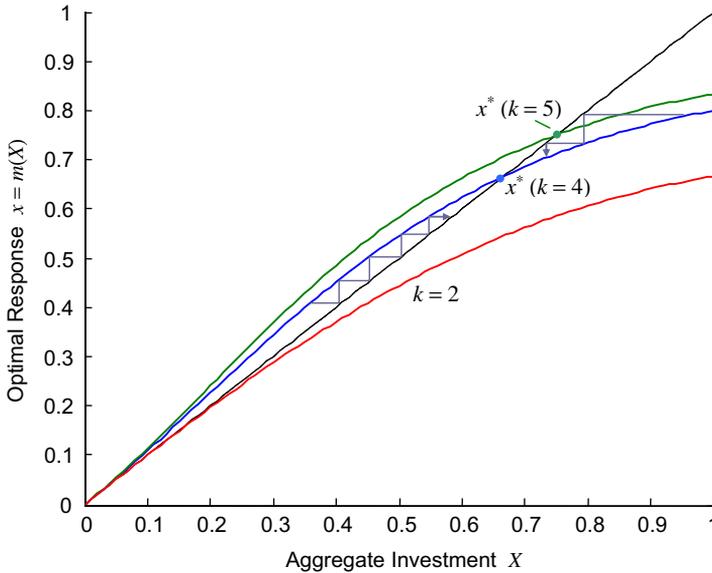


Fig. 1. Reaction function  $m(X)$  when the technology has zero expected return, risk aversion  $\gamma = 3$ , and potential payoff  $k = 2, 4, 5$ . Only equilibria  $x^* = m(x^*)$  with positive investment are stable; such equilibria exist if and only if  $k > 2$ , and the level of investment  $x^*$  increases with  $k$ .

Fig. 1 illustrates our results showing the reaction function for  $k = 2, 4$ , and  $5$ . Note that in each case,  $m(0) = 0$  and no investment in the risky technology is an equilibrium (which is the first-best outcome). For  $k = 2$ , there is no equilibrium with positive risky investment. When  $k = 4$  or  $5$ , there exists an equilibrium with positive risky investment, and the level of investment increases with  $k$ . Finally, we illustrate the convergence of the mapping  $x_{i+1} = m(x_i)$  for the case  $k = 4$ .

#### 4.2. General risk aversion

The results with  $\gamma = 3$  are clear and striking. For a general  $\gamma$ , a closed form solution is no longer available. Nevertheless, using the reaction function we are able to prove that the main conclusions still hold. Provided that the risk aversion coefficient is not too low, investment in the risky technology is increasing in its risk. Also, if the technology is not risky enough it does not attract any investment. Formally:

#### Proposition 4.

- (i) For  $\gamma > 3$ ,
  - a.  $x = 0$  is unstable.
  - b. There exists a stable equilibrium with positive investment in the risky technology ( $x > 0$ ). In that equilibrium, investment in the risky technology is increasing in  $k$  provided that  $k > k^* = \sqrt{2} + 1$ .
- (ii) For  $3 \geq \gamma > 2$ , for  $k$  large enough there exists an equilibrium with  $x > 0$ , whereas for  $k < 1 + 1/(\gamma - 1)$ , then  $x = 0$  is the unique equilibrium.

The first result follows from the fact that  $m'(0) > 1$  when  $\gamma > 3$ . Because  $m(0) = 0$  and  $m(1) < 1$ , there is a stable equilibrium with positive investment in the risky technology, while zero investment is unstable. Again the equilibrium with positive investment in the risky technology is Pareto dominated by the equilibrium in which the risky technology is not utilized.

## 5. Productive investment and competition

In the last section, the risky technology had zero expected return and constant returns to scale. In addition to its tractability, this case had the advantage that the first-best level of investment is zero, making clear our result regarding overinvestment. However, a weakness of this case is that we are forced to rely on an equilibrium selection criterion to rule out the efficient outcome. In this section we look at a setting in which some amount of investment in the risky technology has a positive expected return. We find that here the equilibrium outcome is unique. As before, the degree of overinvestment increases with the risk of the technology. Moreover, if the technology has decreasing aggregate returns to scale (due, for example, to competition between producers), investment may be sufficiently excessive so that in equilibrium the technology has a negative expected return.

### 5.1. Investment with a positive expected return

Suppose  $\pi k = \mu > 1$ . In this case, the risky technology has a positive expected return and  $\alpha < 1$ . From (4), the reaction function  $m(x)$  increases as  $\alpha$  decreases. As a result,  $m(0) > 0$  and there is a unique equilibrium. In Fig. 2 we illustrate the equilibrium in terms of the reaction function when the investment has an expected return of 25%. The equilibrium corresponds to the unique fixed point  $m(x^*) = x^*$ .

Note from (4) that the first-best level of investment can be computed from the reaction function when  $X = 0$ :

$$m(0) = \frac{h(1) - h(1)\alpha}{h(1) + (k-1)h(1)\alpha} = \frac{1 - \alpha}{1 + (k-1)\alpha} = x_b.$$

Because  $x^* = m(x^*) > m(0)$ , in equilibrium there is overinvestment, as can be seen in the figure. Also note that, as in the prior analysis, overinvestment is larger for the higher risk technology (whereas first-best investment declines).

Thus, when the investment has positive expected return, we obtain a unique equilibrium outcome in which agents overinvest in the risky technology. Note, however, that because the technology has a positive expected return, as an outside observer one cannot distinguish whether the equilibrium involves overinvestment due to relative wealth effects, or whether investment is first-best but agents are much less risk averse. As a result, we cannot unambiguously label the outcome as an investment “bubble.” In fact, in this case one could argue that relative wealth effects serve to offset producers’ risk aversion, stimulating investment and making the economy more productive as a result.

### 5.2. Competition and negative expected returns

While relative wealth concerns may stimulate additional productive investment, they may also induce destructive competition between producers. Thus far, the returns to the

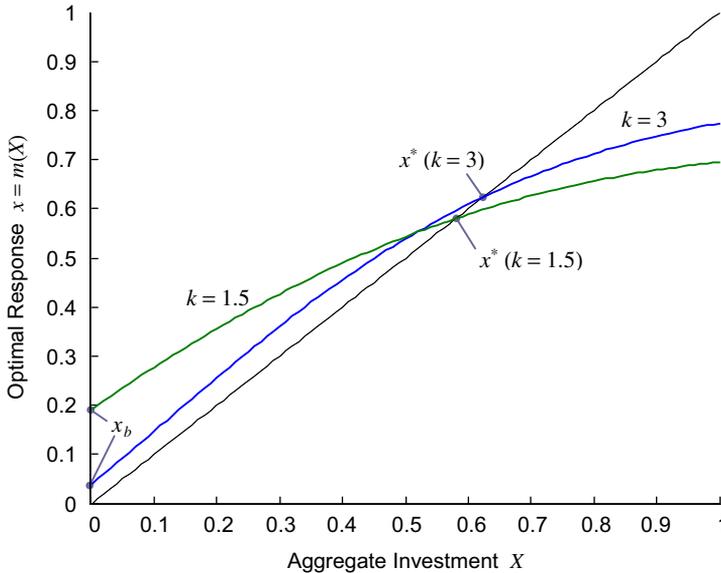


Fig. 2. Reaction function  $m(X)$  with an expected return of 25% ( $\mu = 1.25$ ), risk aversion  $\gamma = 3$ , and potential payoff  $k = 1.5, 3$ . In this case there is a unique equilibrium  $x^*$  with overinvestment, and the degree of overinvestment is larger when the technology has higher risk ( $k$ ). Note that the first-best level of investment,  $x_b = m(0)$ , decreases with  $k$ .

risky investment technology have been independent of the amount invested. In this section we consider the case in which there is competition between producers and/or decreasing returns to scale in the technology. Changing the technology in this way has several important consequences:

- There is a unique equilibrium, and therefore no need for an equilibrium selection criterion. This equilibrium results in positive investment in the risky technology.
- With relative wealth concerns, producers may compete away all surplus and in equilibrium the risky investment can have a negative expected return.
- Investment in the risky technology is increasing, and the expected return is decreasing, with the risk of the technology.

Thus, with decreasing aggregate returns to scale, we obtain a unique outcome with bubble-like investment behavior.

*Technology.* Suppose that if the technology is successful, output is  $k(X) = \min(\bar{K}, K/X)$  for constants  $\bar{K} > K$ . The constant  $\bar{K}$  represents the maximum output per unit invested, which is achieved as long as the total output,  $\bar{K}X$ , does not exceed  $K$ . The amount  $K$  represents the maximum *total* output (we can think of this amount as the total size of the market for the new technology). For  $\bar{K}X > K$ , or equivalently  $X > X^K \equiv K/\bar{K}$ , investment is unproductive. Total output remains equal to  $K$ , and each producer's investment determines only his share,  $x/X$ , of the market. This setting fits a story often told regarding a new technology: while the technology itself is productive, there is excessive entry and producers end up competing away the profits. (For example, while the internet is a useful

technology, competition prevented many of the initial online retailers from generating profits.)

The probability of the technology’s success is  $\pi$ , and we assume that  $\pi\bar{K} > 1$ . Thus, for  $X \leq X^K$ , the technology has a positive expected return of  $\pi\bar{K} - 1$ . For  $X > X^K$ , the equilibrium expected return of the technology is  $\pi K/X - 1$ , which falls with  $X$  and is positive if  $X < X^0 \equiv \pi K$ , and negative if  $X$  exceeds  $X^0$ .

*Equilibrium.* With relative wealth concerns, the optimal investment  $x$  for an individual producer given aggregate investment  $X$  can be found by replacing  $k$  with  $k(X)$  in (4):

$$x = m(X) = \frac{h(1 + X(k(X) - 1)) - h(1 - X)\alpha(X)}{h(1 + X(k(X) - 1)) + (k(X) - 1)h(1 - X)\alpha(X)}, \tag{5}$$

where  $\alpha(X) = ((1 - \pi)/\pi(k(X) - 1))^{1/\gamma}$ . An equilibrium is a fixed point  $x^* = m(x^*)$ .

As before,  $m(0)$  is the first-best level of investment given output  $k(0) = \bar{K}$ . Because investment is unproductive beyond  $X^K$ , the first-best outcome is<sup>11</sup>

$$x_b = \min(m(0), X^K) > 0.$$

With relative wealth concerns, investment will always exceed the first best ( $x^* > x_b$ ). Moreover, the following proposition provides sufficient conditions for  $x^* > X^0$ , so that relative wealth concerns lead producers to compete away all profits, and investment in the risky technology has negative expected returns.

**Proposition 5.** *If  $\gamma \geq 3$  and  $K^{\gamma-2} > 1/(1 - \pi K) + 1$ , then investment in the risky technology satisfies  $x^* > X^0$  and the risky technology has a negative expected return.*

We illustrate the possibility of an equilibrium with negative expected returns in Fig. 3. We set  $\gamma = 3$  and consider three examples, with maximum output of  $K = 1, 2,$  or  $4$ . In each case,  $\bar{K} = 4K$  and  $\pi = 0.5/K$ . Thus, for  $X < X^K = 0.25$ , the technology has expected return  $\pi\bar{K} - 1 = 100\%$ , and the expected return drops to 0 at  $X^0 = \pi K = 0.5$ . Note that in all three cases, the technology’s expected return is the same for each level of investment  $X$ , while its risk increases with  $K$ .

In this example, the sufficient condition in Proposition 5 becomes  $K > 3$ . As Fig. 3 shows, in equilibrium the technology has a negative expected return ( $x^* > X^0 = 0.5$ ) when  $K = 4$ , but also when  $K = 2$ , indicating that the sufficient condition in the proposition is stronger than necessary. When  $K = 1$ , overinvestment reduces profits, but does not eliminate them entirely.

Note that as before, investment increases with the risk of the technology. This result is in contrast with the first-best outcome,  $x_b = m(0) < X^K = 0.25$ , which decreases with  $K$ .<sup>12</sup>

### 6. Relative wealth concerns and investment

In our setting, relative wealth concerns arise from the fact that producers compete with each other for services in period 2, leading to the indirect utility function  $V$  that depends on own wealth and aggregate wealth. Alternatively, we could simply assume a utility function

<sup>11</sup>If  $m(0) > X^K$ , then even in a setting without trading frictions and relative wealth concerns, investment will exceed  $x_b$  due to the negative externality that producers impose on each other by competing for market share. If we let  $x_c$  be the competitive solution in this case, then by analogy with (1),  $x_c$  solves  $x_c = (1 - \alpha(x_c))/(1 + (k(x_c) - 1)\alpha(x_c))$ , and  $x_c = x_b$  unless  $m(0) > X^K$ , in which case  $x_b < x_c < X^0$ . A proof identical to that of Proposition 1 implies that due to relative wealth effects,  $X^* > x_c$ .

<sup>12</sup>Because  $m(0) < X^K = 0.25$ , we also have that  $x_c = x_b = m(0)$  in this example (see fn. 11).

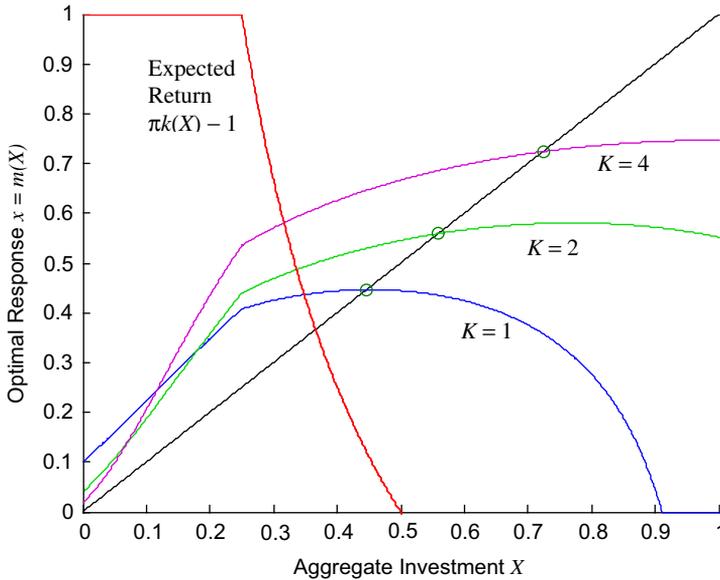


Fig. 3. Reaction function  $m(X)$  when producers compete. In the example, risk aversion  $\gamma = 3$ , maximum total output  $K = 1, 2, \text{ or } 4$ , maximum payoff  $\bar{K} = 4K$ , and success rate  $\pi = 0.5/K$ . Note that the equilibrium is unique and, for  $K = 2$  or  $4$ , entails negative expected returns.

in which agents care directly about their relative wealth. In this section we discuss the properties of agents’ preferences over relative wealth that lead to the possibility of bubbles.

Given the utility function  $V(y, Y)$ , where  $y$  is the agent’s own wealth and  $Y$  is the average wealth in the community, Dupor and Liu (2003) describe the following notions of relative wealth effects:

- *Jealousy*: The agent’s utility is decreasing in average community wealth,  $V_2 < 0$ .
- *Keeping Up with the Joneses (KUJ)*: The agent’s marginal utility is increasing in average community wealth,  $V_{12} > 0$ .

In our setting, with  $V$  defined as in (2), agents exhibit jealousy because  $h(Y)^\gamma/(1-\gamma)$  is decreasing in  $Y$ . In addition, if  $\gamma > 1$  then  $h(Y)$ , and thus marginal utility, is increasing in  $Y$  and agents try to “keep up with the Joneses.”

Dupor and Liu demonstrate that if  $V$  exhibits jealousy, then due to the negative externality, agents overconsume in equilibrium relative to the social optimum. We are interested in the consequences of relative wealth concerns for investment. Let  $x$  be the investment in a risky technology with excess return  $\theta$ , which is a random variable. Agents choose  $x$  to solve

$$x = m(X) = \arg \max_x E[V(1 + x\theta, 1 + X\theta)],$$

where  $X$  is the aggregate investment in the community. Then,

$$\frac{\partial}{\partial X} m(X) = - \frac{E[V_{12}\theta^2]}{E[V_{11}\theta^2]}. \tag{6}$$

Thus,  $x^*$  is increasing in  $X$  if preferences satisfy KIJ. The intuition for this result is that because agents' marginal utility for income is higher when their community is wealthy, they choose to align their investment decisions more closely to their community members' decisions so as to be wealthy when others are. In other words, the fear of being left behind makes agents act more like their neighbors.

The result that  $m'(X) > 0$  is sufficient to produce overinvestment when  $m(0) = x_b > 0$ . Thus, KIJ is a sufficient condition for overinvestment when investment occurs. Some of our examples, however, demonstrate the even stronger result that overinvestment can occur even when the expected return on investment is nonpositive, so that no investment would occur absent relative wealth concerns. In these examples, producers invest simply because others do so. When  $m(0) = 0$ , a sufficient condition for such an equilibrium to exist is  $m'(0) > 1$ . This condition requires a strengthening of KIJ, which we call the herding property:

- *Herding*: The KIJ effect dominates the agent's risk aversion,  $V_{12} > -V_{11}$ .

When preferences exhibit the herding property, the incentive for agents to follow each other is sufficiently strong that it may be self-sustaining in equilibrium. In particular, agents may adopt investment policies that are excessively risky simply because others are making similar investments. In our model, the herding property holds when  $\gamma \geq 3$ .

Ultimately, we are agnostic regarding the source of relative wealth concerns. However, we believe that in reality both sources are likely: individuals care directly about their relative standing, and they face price effects from competition over scarce resources. We focus in this paper on the rational setting with  $V$  given by (2) to fix ideas and avoid an ad hoc specification of the utility function.

## 7. Public versus private firms

For simplicity, in the model thus far there is no separation between control and ownership. As a result we assume that firms maximize a utility function rather than market value. Such a framework best describes privately held firms rather than public firms. In this section we show how our model can be extended to include public firms. We briefly describe two alternative specifications of the model that lead to the same results but have different empirical interpretations.

Consider first replacing our "producers" with investors who are endowed with and trade shares in a large number of publicly traded, price-taking firms. The firms produce goods and choose an investment policy to maximize their share price. All agents have the same utility function over goods and services as before. We assume that financial markets are complete, but that service providers do not participate in financial markets. Because only investors are shareholders, and firms are competitive, the objective of maximizing the firm's share price is unanimously supported. Prices for output in state  $Y$  are given by the marginal utility of the representative investor,  $V_1(Y, Y)$ , so that the firms' value maximization problem is given by

$$\max_x E((1 + x\theta)V_1(Y, Y)). \quad (7)$$

The first-order condition is given by

$$E(\theta V_1(Y, Y)) = 0.$$

This yields the same equilibrium as we derived previously.

A second alternative specification of the model is as a model with three goods, namely, capital, which serves as the numeraire, services, and a consumption good. All agents maximize a CRRA utility function with risk aversion coefficient  $\gamma$  over the consumption good, which is consumed at the second date. Finally, there are two types of agents, agents who are endowed with services and do not participate in financial markets and agents who are endowed with and trade claims to firms that possess capital.

At the first date, firms decide which technology to invest their capital in. Given an investment of  $x$  in the risky technology, firms have  $(1 + x\theta)$  “units” of capital in the second period that they can then use to produce goods. (We can interpret the units of capital as the firm’s productivity, rather than a physical quantity.) At the second date, capital and services are used to produce the consumption good according to a constant elasticity of substitution (CES) production function:

$$f(c_g, c_s) = \left( c_g^{1-\gamma} + c_s^{1-\gamma} \right)^{1/(1-\gamma)}.$$

Firms must purchase services from service providers in the second period in order to produce. Again, firms make their first-period investment decision and second-period production decision to maximize their share price.

With this choice of production and utility functions, this alternative specification leads to the same outcomes as in the initial model. Now, firms overinvest in the risky technology out of concerns regarding the price of labor services that will arise in the future.

## 8. Conclusions

In this paper we demonstrate that rational endogenous relative wealth considerations lead to herding in real investment decisions, and can be an important factor supporting the creation of technology bubbles.<sup>13</sup> Our main result shows that not only do relative wealth considerations lead to the overadoption of risky production technologies, but *in equilibrium* the rate of adoption and the use of these technologies can be increasing in their riskiness.

We derive this result using a stylized yet fully rational general equilibrium model with two key ingredients. First, investors’ utility functions depend only on their own consumption. Second, there are scarce consumption goods whose price increases with the wealth of investors. As a result, agents’ ability to consume depends on their relative wealth. This externality induces a herding incentive: agents choose to make investment decisions that are similar to those of the rest of the population to avoid being poor when their cohort is wealthy. If agents are sensitive to the wealth of others, making different investment decisions than the crowd increases the risk of their relative wealth. The riskier the technology, the greater is agents’ concern for being left behind, and the stronger the herding effect.

<sup>13</sup>While we produce real investment bubbles in this setting, one may wonder if pure security price bubbles could also occur. That is, would we see similar effects in security prices if real investment were fixed? In a simple model with one community, the answer is no. Because the aggregate supply of securities is fixed, herding will distort prices upwards, and this price increase will offset the incentives to herd. In a production economy, since supply is not fixed, there are no such price effects and there is the potential for herding effects in equilibrium. Bubbles can also occur in security markets if there are multiple communities that can trade with each other, as in DeMarzo, Kaniel, and Kremer (2006).

Our results provide a rational explanation for overinvestment in risky technologies. We note, however, that not all risky technologies can be expected to lead to overinvestment, as the incentive to overinvest is with respect to aggregate rather than idiosyncratic risk. It is those technological innovations that have an impact on the economy at large and hence that have highly correlated investment opportunities for which the incentives we document are likely to emerge.

## Appendix

### A.1. Proof of Proposition 2

We begin by considering producers. Denote by  $Y$  the date-2 producers' individual/aggregate endowment of goods. Since the endowment may depend on the state, it is a random variable. Given  $1/(1-\gamma) < 0$ , we need to show that

$$E\{Y^{1-\gamma}h(Y)^\gamma\} > h(1)^\gamma.$$

In equilibrium  $Y^{-\gamma}h(Y)^\gamma$  is equated across the two states and hence it equals some constant  $c$ . Given that the risky technology yields zero return we conclude that  $E(Y) = 1$ . Hence, we just need to verify that

$$c^{1/\gamma} = Y^{-1}h(Y) > h(1).$$

Multiplying both sides by  $Y$  and taking expectations we only need to verify that

$$E(h(Y)) > E(Y)h(1).$$

The above follows from the convexity of  $h$  for  $\gamma > 2$  and the fact that  $E(Y) = 1$ .

We now consider service providers. Because the price of the service is given by  $Y^\gamma$ , we need to show that

$$E\{Y^{\gamma(1-\gamma)}h(Y)^\gamma\} > h(1)^\gamma.$$

Since  $\gamma > 1$ , it is sufficient to show that

$$E\{Y^{1-\gamma}h(Y)\} > h(1).$$

This above holds since  $Y^{1-\gamma}h(Y) = Y^{1-\gamma} + 1$  is again convex.

### A.2. Proof of Proposition 4

(i) As mentioned in text, we use  $m(X)$  to denote the optimal investment of an individual that faces an aggregate investment in the risky technology of  $X$ . Existence of a solution with  $x > 0$  follows from the fact that given  $\pi k = 1 = \alpha$ ,  $m(0) = 0$ , and

$$m'(0) = \frac{\gamma - 1}{2},$$

so that for  $\gamma > 3$ ,  $m'(0) > 1$ , and for  $k \geq 1$ ,  $m(1) < 1$ . The fact that  $m'(0) > 1$  also implies that  $x = 0$  is unstable.

The derivative of  $m(x)$  with respect to  $k$  is proportional to

$$(1 + (k - 1)x)^{\gamma-2}(((\gamma - 2)k + 1)x - 1) + (1 - x)^{\gamma-1}. \quad (8)$$

Thus, a sufficient condition for the derivative to be positive is that

$$x > \frac{1}{(\gamma - 2)k + 1}. \tag{9}$$

This condition holds at the solution when  $\gamma = 3$ , since

$$\frac{k - 2}{k - 1} > \frac{1}{(3 - 2)k + 1},$$

is satisfied for  $k > k^* = \sqrt{2} + 1$ . Now, a direct computation shows that for  $\gamma > 1$  and  $k > 1$ ,  $m(x)$  is increasing in  $\gamma$ . Therefore, for  $\gamma \geq 3$ , (9) holds at  $x^*$  when  $k > k^*$ , which yields the proof.

(ii) We first note that  $m(1) < 1$ . A direct computation shows that for  $k > 1$ , for  $0 < x < 1$ ,  $m(x) > x$  is equivalent to

$$(1 - x)((1 + (k - 1)x)^{\gamma - 1} - (k + 1 - x)(1 - x)^{\gamma - 2}) > k.$$

For  $x \leq 0.5$  a sufficient condition is

$$(1 + (k - 1)x)^{\gamma - 1} - (k + 1 - x) > 2k,$$

or alternatively

$$(1 + (k - 1)x)^{\gamma - 1} > 3k + 1 - x.$$

Because  $\gamma > 2$  the above holds for  $k$  large enough. Thus,  $m(x) > x$  for some  $x$  and there exists an equilibrium with  $x > 0$ .

To show that no equilibrium with positive investment exists if  $k$  is sufficiently small, it suffices to show that

$$(1 - x)(1 + (k - 1)x)^{\gamma - 1} < k.$$

Note that at  $x = 0$  this inequality holds for any  $k > 1$ . Let  $k = 1 + \varepsilon$  so that this condition becomes

$$(1 - x)(1 + \varepsilon x)^{\gamma - 1} < 1 + \varepsilon.$$

Taking derivative of the left-hand side with respect to  $x$  yields

$$(1 + \varepsilon x)^{\gamma - 2}((1 - x)(\gamma - 1)\varepsilon - (1 + \varepsilon x)),$$

which is negative if

$$((1 - x)(\gamma - 1) - x)\varepsilon < 1.$$

A sufficient condition for this inequality to hold is  $\varepsilon < 1/(\gamma - 1)$ , which is equivalent to  $k < 1 + 1/(\gamma - 1)$ .

### A.3. Proof of Proposition 5

To prove the proposition it suffices to show that for every  $X \leq \mu = \pi K$ ,  $m(X) > X$ .

For  $\mu \geq X > K/\bar{K}$ :

Note that  $m$  is increasing in  $\alpha(X)$ , and for  $X > K/\bar{K}$ ,  $\alpha(X)$  is increasing in  $X$ . Combining this with the fact that  $\alpha(\mu) = 1$  and plugging into (5) implies that

$$m(X) > \frac{h(1 + X((K/X) - 1)) - h(1 - X)}{h(1 + X((K/X) - 1)) + ((K/X) - 1)h(1 - X)}.$$

Simple algebra shows that ensuring that the right-hand side of (2) is larger than  $X$  is equivalent to

$$[1 + (1 + K - X)^{\gamma-1}](1 - X) > [1 + (1 - X)^{\gamma-1}](1 + K - X).$$

Using the fact that  $(1 + K - X)^{\gamma-1} \geq (1 - X)^{\gamma-1} + K^{\gamma-1}$  we replace  $(1 + K - X)^{\gamma-1}$  with  $(1 - X)^{\gamma-1} + K^{\gamma-1}$ , implying that the condition holds if

$$K^{\gamma-2} > \frac{1}{1 - X} + (1 - X)^{\gamma-2},$$

which holds when  $K^{\gamma-2} > 1/(1 - \mu) + 1$ .

For  $X \leq K/\bar{K}$ :

In this case Eq. (5) becomes

$$m(X) = \frac{h(1 + X(\bar{K} - 1)) - h(1 - X)\hat{\alpha}}{h(1 + X(\bar{K} - 1)) + (\bar{K} - 1)h(1 - X)\hat{\alpha}},$$

where

$$\hat{\alpha} = \left( \frac{1 - \pi}{\pi(\bar{K} - 1)} \right)^{1/\gamma} < 1.$$

Simple algebra shows that  $m(X) > X$  is equivalent to

$$\frac{1 - X}{1 + (1 - X)^{\gamma-1}} \frac{1 + (1 - X + X\bar{K})^{\gamma-1}}{1 - X + X\bar{K}} > \hat{\alpha}$$

which holds if  $\gamma \geq 3$  and  $\bar{K} > 1$ .

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