Impact of Managerial Commitment on Risk Taking with Dynamic Fund Flows

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ABSTRACT

We present a model with dynamic investment flows, where fund managers have the ability to generate excess returns and study how forcing them to commit part or all of their personal wealth to the fund they manage affects fund risk taking. We contrast the behavior of a manager that may invest her personal wealth in a private account to a manager that is either forced to commit her wealth to the fund she manages, or a manager who is not allowed to hold risky assets held by the fund privately. We show that a fund managed by a manager with higher ability does not necessarily achieve higher expected returns but achieves lower idiosyncratic volatility. For a manager with constant ability, restrictions placed on her personal account do not influence her choices in the fund, while for a manager whose ability varies stochastically they result in higher expected returns and idiosyncratic volatilities. Fund strategies can be non-monotone both in manager's commitment level and the ratio of manager to investor wealth. Our results are robust in the case of incomplete information and in the case of competing managers with correlated ability.

1. Introduction

Delegated investment management has grown considerably over the last 50 years, largely as a result of greater availability of individual retirement accounts. While part of this increase was satisfied by the introduction of index funds, it also resulted in a greater number of actively-managed, performance-driven funds becoming available to retail investors. Along with the greater importance of delegated investment management came increased scrutiny of the personal investment decisions of fund managers. For mutual funds, the Securities and Exchange Commission (SEC) introduced rules 17j in 1970 and 17j-1 in 1980 (subsequently amended in 1999) to regulate certain investment practices by portfolio managers.[1] Although personal investment by portfolio managers is not currently banned, rule 17j-1 requires establishing a code of ethics and strict reporting requirements for mutual fund managers and employees regarding transactions in their personal accounts.[2]

An examination of the code of ethics of all the major fund complexes, measured by assets under management, shows that certain features are prevalent and explicitly intended to limit conflict of interests: prohibiting trading against fund positions; mandatory blackout periods on trading in a manager's personal account before and after a fund trades in a security; pre-clearance from the compliance department or the code of ethics committee for trading any covered securities; extreme restrictions up to flat out prohibition on purchasing IPOs, purchasing private placements, trading options and short selling; ban on short-swing (typically 60 days) trading where profits obtained are relinquished; and ban on trading while in possession

[1] An event driving the increased scrutiny was the firing in January 1994 of the portfolio manager of the largest fund in the INVESCO family, charged with violating the company's internal procedures for personal trading. Following the firing the Investment Company Institute (ICI) established an advisory group that published in May 1994 a report with guidelines for personal investing by portfolio managers, as well as other mutual fund employees.

[2] In addition to rules 17j and 17j-1, a separate rule, 204A-1, applies to fund advisers. The rule requires certain supervised persons, called "access persons," to report their personal securities transactions and holdings. According to the rule, "an access person is a supervised person who has access to nonpublic information regarding clients' purchase or sale of securities, is involved in making securities recommendations to clients or who has access to such recommendations that are nonpublic. A supervised person who has access to nonpublic information regarding the portfolio holdings of affiliated mutual funds is also an access person."

of material nonpublic information about an issuer of a security. To better monitor the manager's actions in their personal account, some fund complexes require that all the manager's accounts be maintained with the complex, for example Fidelity.[3]

In this paper we examine the impact managerial ability and different types of restrictions on the personal accounts of portfolio managers may have on fund portfolios. Regulatory authorities are concerned with preventing activity detrimental to shareholder interests. Our focus is on one such activity: fund portfolio distortions that emanate from restrictions imposed on the fund portfolio manager. Similar to the restrictions considered in the ICI report by Fiumefreddo, Fossel, Graham, Lynch, Pozen, and Riepe (1994), we consider: a) forcing the manager to hold a personal portfolio that is identical to the fund portfolio, only allowing the manager to invest in the fund;[4] b) restrictions that preclude the manager from investing in assets held by the fund; and, c) intermediate cases, where a percentage of the manager's wealth is invested in the fund portfolio. The restrictions we consider are reflected in the practices across funds: Evans (2008) studies domestic equity funds run by a single manager, and finds significant cross-sectional differences across funds in the amount of the managers own money that invested in the fund she runs.[5] While for close to 50% of funds the manager does not invest in the fund she runs, for 20% the manager invests at least a million dollars in the fund.

Our framework considers a manager of an actively-managed fund that has the ability to generate excess returns beyond those available to retail investors by taking idiosyncratic risk.[6]

[3]The fund complexes we considered are Fidelity, Vanguard, T.Rowe price, Oppenheimer, American Funds, Franklin Templeton, Dodge and Cox, and Columbia Management.

[4]This restriction mimics the behavior of hedge fund managers. For mutual funds, this restriction was described by Richard Y. Roberts, commissioner, U.S. Securities and Exchange Commission, as forcing the managers to "eat their own cooking" — D.C. Bar and George Washington University Merging Financial Markets Conference, Washington, D.C., March 25, 1994, as reported on page 19 of the ICI report.

[5]We use gender to distinguish between a manager (female) and an investor (male) throughout the paper.

[6]There is considerable debate in the literature on whether fund managers are able to consistently generate positive excess returns. In this paper we start from the assumption that managers can achieve positive excess returns and focus on the normative implications constraints on the managers have on fund allocations, expected returns, and idiosyncratic volatility.

In our base case we consider a manager who can achieve a constant Sharpe ratio, expressed as an investment opportunity available only to her

We start by analyzing the relation between the level of managerial ability to a fund's expected excess return and idiosyncratic volatility. We find that fund expected returns are largely independent of ability, while idiosyncratic volatility decreases as managerial ability increases. What drives this result is the change in managerial behavior when ability changes: managers with higher levels of ability can choose fund allocations that achieve the same expected returns at lower levels of idiosyncratic volatility. The increase in the fund's Sharpe ratio induces the investor to increase his investment to the fund, leading to higher fees for the manager.

Next we relate fund allocations, expected returns and idiosyncratic volatility to restrictions placed on fund managers. We show that the changes are greatest for managers that have to commit their entire wealth to the fund they manage. Since managers with constant ability do not face significant hedging demands, due to the constant opportunity set, fund allocations are largely unchanged as restrictions are placed on the managers, with only managers that commit their entire wealth to the fund and who are relatively wealthy compared to the size of the fund tilting fund allocations based on their own preferences.

Fund allocations are determined by the interaction of two factors: the desire of the manager to extract rents from the investor in exchange for access to the manager's investment ability; and the manager's personal preferences which enter the determination of the fund portfolio as a result of the restrictions on the manager's personal account. We show that the interplay between the two can lead to fund allocation being non-monotone in the ratio of manager to fund wealth and that this non-monotonicity depends also on how the current investment opportunity compares to the long-run investment opportunity. Furthermore, with restrictions on the manager's portfolio, allocations can depend non-monotonically on the fraction of the wealth the manager commits to hold in the fund.

We provide wealth equivalents, i.e., the percentage change in the wealth of a manager that she would require in order to be indifferent between facing an additional restriction and not facing the restriction, that capture the effective cost of restricting a manager relative to a manager that is able to utilize the fund strategy in her personal account. The wealth equivalents are non-monotonic with the level of managerial ability for managers that commit a significant fraction of their wealth to the fund – an indication that for low and high levels of ability a committed manager is not able to adjust her exposure as well as for intermediate levels of ability.[7]

In an extension of our base case, we consider a manager with stochastic, mean-reverting, ability, expressed as an investment opportunity with a mean-reverting Sharpe ratio. When managers have stochastic ability, their hedging demand is greater, which, in combination with the fact that restrictions are placed on the allocation of their personal wealth, limits their flexibility of satisfying this hedging demand in their personal account, and leads to different fund allocations. In our calibrated examples, the overall effect is to choose allocations with higher expected returns and idiosyncratic volatilities.

We also provide comparative statics for different levels of fees, managerial risk aversion, and, for the case of a manager with composite ability, correlation between the factors describing the components of managerial ability. The results are consistent with the intuition we have already described.

In addition to studying cases where managerial ability is known, we verify that similar results hold when there is incomplete information about managerial ability, so that both investors and the manager jointly learn about the manager's ability from observing realized fund returns.

To determine the robustness of our results, we also consider a case of two competing managers. Competition has the potential to alleviate the problem of misalignment of incentives between the managers and the investor, by inducing the managers to cater to the investor's desires in order to attract his funds. We allow for different levels of ability between managers

[7]Similar calculations of wealth equivalents for the investor, show that the effective cost of facing an unrestricted manager is relatively small.

and for correlation between the managers' opportunity sets. We find that when correlation between the managers' opportunity sets is zero, our results for the case of a single manager carry through to the case of two managers: each manager acts as a monopolist. When correlation is non-zero but strictly different from ± 1, the managers do compete, but the results remain similar to the case of a single manager. The imperfect correlation translates to each manager having some monopoly power that she exercises by somewhat limiting access to her ability, forcing the investor to increase his allocation to the fund. For managers with correlated but differential ability, competition translates to the manager with the lower level of ability compensating for the lower ability by making more of her ability available through her fund to the investor.[8] The comparative static for differential ability for idiosyncratic volatility and Sharpe ratio for the returns of a fund in the case of a single manager applies to the case of competing managers: the fund managed by the manager with the higher level of ability has lower idiosyncratic volatility and higher Sharpe ratio than the fund managed by the manager with lower ability.

The remainder of the paper is organized as follows: Section 2 reviews the literature. The model and a closed-form solution for the objective of maximizing investor participation is presented in Section 3. Section 4 presents numerical results for the fund allocations chosen by managers of different levels of ability, commitment, risk aversion, fund fee rates, and when allowing learning managerial ability by observing realized returns. Section 5 describes an extension of our model that considers the case of two managers that compete with each other. It is followed by the final section which concludes. The appendices discuss the homogeneity of the value function, the numerical method, and details of the learning about managerial ability and competition setups.

[8] Competition results in the manager with higher levels of ability making more of her ability available through the fund, compared to the case of a single manager. However, the effect is larger for the manager with the lower level of ability.

2. Literature Review

A significant part of prior research on fund managers' dynamic portfolio choice decisions has focused on risk shifting implications when managers' compensation depends explicitly on fund performance relative to a benchmark (e.g. Carpenter (2000) and Cuoco and Kaniel (2011)), or when terminal date flows are driven by an exogenously specified performance-flow relation relative to a benchmark (e.g. Basak, Pavlova, and Shapiro (2007) and Chen and Pennacchi (2009)). Impact of high-water mark incentive schemes is considered, for example, in Goetzmann, Ingersoll, and Ross (2003), Panageas and Westerfield (2009), Lan, Wang, and Yang (2013), and Drechsler (2014).[9] We instead focus on the fraction of assets under management contract, and implications for the optimally determined inter-temporal flows.

While the literature on the allocation choices of funds is enormous, to our knowledge, little is known about the effect of managerial ability and restrictions placed on the personal accounts of managers on fund allocations. Khorana, Servaes, and Wedge (2007) find that increased commitment by mutual fund managers to their fund is a predictor of higher expected returns, while Evans (2008) shows that the level of managerial participation is positively associated with style adjusted returns and negatively associated with fund turnover. This result appears to be driven by asymmetric information on the manager's ability between fund shareholders and managers: better managers choose to hold more of their wealth in the fund. While our framework does not account for asymmetric information or for managerial discretion regarding commitment levels, we discuss how restrictions placed on managers can impact fund performance. Pool, Stoffman, Yonker, and Zhang (2014) consider shocks to the personal wealth of fund managers, based on the reduction of house prices during the 2007-2008 crisis. They find that fund managers who suffered the greatest losses subsequently reduce both the systematic and idiosyncratic risk of fund portfolios.

[9] Dai, Jin, and Liu (2011) analyze the optimal trading strategy of a fund facing position limits and differential liquidity of its assets.

The bulk of the pre-existing literature completely restricts a manager's ability to trade on her own account, implicitly assuming that all fund manager wealth is tied to the proceeds of her compensation for managing the fund. We depart from this literature by focusing instead on understanding how requiring managers to commit part of their wealth to the fund they manage affects fund allocations, for differing levels of commitment.

A paper that is closely related to ours is Hugonnier and Kaniel (2010). Hugonnier and Kaniel (2010) allow the manager unrestricted use of her personal account to hedge her risky asset exposure through the fund. In contrast, our focus is on the effect that restrictions on the manager's personal investment account have on fund allocations, expected excess returns and idiosyncratic volatility.

While most prior literature analyzing fund managers' allocation decisions ignores intertemporal dynamic fund flows or assumes they are driven by an exogenously specified functional form (e.g. Vayanos (2004), and Kaniel and Kondor (2013) — exceptions include Hugonnier and Kaniel (2010), Vayanos and Woolley (2013) and He and Krishnamurthy (2013)), we account for endogenously optimally determined dynamic investor flows. Vayanos and Woolley (2013) show that a potential contributing factor to momentum and reversal in stock returns are flows between investment funds. In their setting a fund manager determines the fund portfolio and what fraction of her wealth to invest in the fund. The remainder of her wealth is held in the risk free asset. Consequently, the composition of the equity portfolio of the fund and the manager are perfectly aligned. We, on the other hand, also allow the composition of the portfolios to differ.[10] In He and Krishnamurthy (2013), managers are not directly motivated by flows because they do not receive fees based on assets under management. Their main focus is on the amplification of bad shocks through the incentive constraint of managers.

Equilibrium asset pricing implications of delegated portfolio management in a dynamic competitive setting are analyzed in Vayanos (2004), Dasgupta and Prat (2006), Dasgupta

[10] Also, in Vayanos and Woolley (2013), by construction the manager acts as a trading counter party to investors flows.

and Prat (2008), Cuoco and Kaniel (2011), Guerrieri and Kondor (2012), Kaniel and Kondor (2013), Malliaris and Yan (2013), He and Krishnamurthy (2013), Basak and Makarov (2012), Basak and Pavlova (2013). Berk and Green (2004) present a model where competition among skilled managers results in expected returns that are equal across funds, with ability reflected in better managers managing bigger funds. Our results on expected returns and fund size are similar, both for the case of partial equilibrium with a single manager, and the case of competing managers. However, our results differ from the results in Berk and Green (2004) in the idiosyncratic volatility of returns and Sharpe ratios: funds managed by managers with higher levels of ability have lower idiosyncratic volatility and higher Sharpe ratios. The difference stems from the assumption in Berk and Green (2004) that expected returns, and managerial ability, is decreasing in the size of the fund. In our setting expected returns and managerial ability are assumed to be independent of the size of the fund. We are able to endogenously derive our results on expected returns and fund size even with constant returns to scale, because managers adjust the access investors have to managerial ability through the fund in a way that varies with the level of ability, in an effort to extract greater rents from the investor. Our results lead to empirical predictions that we discuss in the conclusions to the paper.

3. Model Description

We consider a partial equilibrium model in which the actions of the market participants do not influence the dynamics of the asset prices. Managerial ability is modeled by an opportunity set available to a manager of a fund that is greater than the opportunity set available to an investor.[11] To keep the model tractable, we represent the investor's opportunity set by a riskless asset.[12] To represent the manager's ability we allow the manager access to both risky and riskless assets. We consider different types of restrictions on the manager's personal invest-

[11] An alternative interpretation is that the investor has limited ability to access the risky asset directly.

[12] An alternative interpretation is that the investor has access to a risky asset, and that the investor's portfolio besides his investment in the fund, serves as a numeraire for the fund portfolio.

ment portfolio: a) we vary the level of commitment of the manager's personal wealth to the fund; b) we forbid the manager from holding the risky assets held by her fund in her personal account, limiting her to the same investment opportunities available to the retail investor outside the fund. In all cases, reflecting practice, we restrict the manager from taking positions in her personal account opposite to the positions taken by the fund. Finally, we impose margin requirements on the manager's personal account and the fund allocations.

Market Dynamics

In line with the literature, we assume that the investment opportunity set available to the retail investor outside the fund is described by a riskless asset that has a constant interest rate r.

$$\frac{dS_{0,t}}{S_{0,t}} = rdt \tag{1}$$

The manager's ability to generate excess returns is represented by allowing her access to additional risky assets. The only way for the investor to benefit from the manager's ability is by investing in the fund. We study different cases, representing managers with different types of ability, where either one or two assets are available. The first case corresponds to a manager with a constant level of ability; i.e., constant Sharpe ratio, represented by a risky asset with returns described by geometric Brownian motion with constant coefficients

$$\frac{dS_{1,t}}{S_{1,t}} = rdt + \sigma_{11}\left(\xi_1 dt + dB_{1,t}\right)$$

where σ_{11} is the volatility, ξ_1 is the Sharpe ratio corresponding to managerial ability, and B_1 is a standard Brownian motion.

We also allow managers with stochastic ability, represented by a second risky asset with returns that follow a stochastic process that depends on two factors. The drift is modeled as the sum of two components, one that is constant and one that is mean-reverting. The second factor of the drift can be thought of as a temporary component of the manager's ability that represents, for example, the dependence of managerial ability on mean-reverting economic factors,

$$\frac{dS_{2,t}}{S_{2,t}} = rdt + \sigma_{21}\left(\xi_1 dt + dB_{1,t}\right) + \sigma_{22}\left(\xi_{2,t} dt + dB_{2,t}\right)$$

where σ_{21}, σ_{22} are constant, B_2 is a standard Brownian motion that is independent of B_1, and ξ_2 follows a CIR mean-reverting process

$$d\xi_{2,t} = \lambda\left(\bar{\xi}_2 - \xi_{2,t}\right) dt + \psi\sqrt{\frac{\xi_{2,t}}{\bar{\xi}_2}} dB_{2,t}$$

where the mean-reversion rate λ, volatility ψ and the long-term mean-reversion level $\bar{\xi}_2$ are constant.[13]

Value of Assets Under Management

For a fund that holds percentages π_1^F, π_2^F of its value in the first and second risky assets, respectively, the dynamics of the value of assets under management, F, is given by

$$\frac{dF_t}{F_t} = \left(1 - \pi_{1,t}^F - \pi_{2,t}^F\right)\frac{dS_{0,t}}{S_{0,t}} + \pi_{1,t}^F \frac{dS_{1,t}}{S_{1,t}} + \pi_{2,t}^F \frac{dS_{2,t}}{S_{2,t}} - \gamma dt \tag{2}$$

where, in order to hold the fund, the investor pays a fee proportional to assets under management, with percentage fee rate γ.[14] The first term of the right hand side of Equation (2) accounts for changes in the value of the fund due to changes in the value of the riskless asset; the second and third terms account for changes in the values of the two risky assets; the last term accounts for the management fee.

[13] Modeling a stochastic opportunity set by assuming a mean reverting Sharpe ratio process is widespread. One of the first papers to make such an assumption was Kim and Omberg (1996).

[14] The fee structure we consider is representative of mutual funds. Given the difficulties in changing the management fees of a mutual fund we consider a fixed fee rate. To increase its fee rate a mutual fund needs to receive approval from its shareholders. Decreasing the fee rate is easier, and funds do sometimes offer reduced fees for a limited amount of time. We provide comparative statics for different fixed rates in Section 4. Allowing different manager compensation structures; for example not based on the size of the fund, but based on comparing performance to a benchmark, see Admati and Pfleiderer (1997), or option-like payoffs common in hedge funds, can lead to different intuition and results.

When the fund invests in risky assets, it has to meet a margin requirement. This requirement is given by

$$m^{+}\sum_{i=1}^{2}\max\left(\pi_i^F,0\right)+m^{-}\sum_{i=1}^{2}\max\left(-\pi_i^F,0\right)\leq 1 \tag{3}$$

where m^+, m^- are the margin requirements for holding positive and negative asset positions, respectively.

Investor Wealth

At each point in time the investor allocates his capital between the fund and the risk-free asset. The dynamics of the wealth of the investor, W^i, is given by

$$\frac{dW_t^i}{W_t^i}=(1-\phi_t)\frac{dS_{0,t}}{S_{0,t}}+\phi_t\frac{dF_t}{F_t} \tag{4}$$

where ϕ is the percentage of the investor's wealth invested in the fund. Investments outside the fund do not incur a fee. We also impose the natural condition that the investor cannot short-sell the fund, $\phi \geq 0$. The first term of the right hand side of Equation (4) accounts for changes in the value of the riskless asset while the second term accounts for changes in the value of the fund.

Manager's Wealth

The manager commits a percentage of her wealth p to the fund and invests the remainder in her personal account. The manager does not pay a fee for her investment with the fund, but the fees paid by the investor are directed to the manager's personal account. Denoting by $W^m, W^{m,F}, W^{m,P}$ the manager's total wealth, wealth invested in the fund, and wealth invested in her personal account respectively, we have that

$$W^m = W^{m,F} + W^{m,P}$$

The proportion of the manager's wealth invested in the fund, p, is given by:

$$p = \frac{W^{m,F}}{W^m}$$

The dynamics of the manager's wealth invested in the fund is given by

$$\frac{dW_t^{m,F}}{W_t^{m,F}} = \left(1 - \pi_{1,t}^F - \pi_{2,t}^F\right) \frac{dS_{0,t}}{S_{0,t}} + \pi_{1,t}^F \frac{dS_{1,t}}{S_{1,t}} + \pi_{2,t}^F \frac{dS_{2,t}}{S_{2,t}} = \frac{dF_t}{F_t} + \gamma dt \tag{5}$$

Equation (5) represents the change in the value of the fund, given in Equation (2), accounting for the fact that the manager does not pay a fee to personally invest in the fund.

Case A: Unrestricted Manager

If the manager is allowed to invest in the same assets as the fund in her personal account, the dynamics of her wealth, $W^{m,P}$ in her personal account is given by

$$\frac{dW_t^{m,P}}{W_t^{m,P}} = \left(1 - \pi_{1,t}^P - \pi_{2,t}^P\right) \frac{dS_{0,t}}{S_{0,t}} + \pi_{1,t}^P \frac{dS_{1,t}}{S_{1,t}} + \pi_{2,t}^P \frac{dS_{2,t}}{S_{2,t}} + \gamma \phi_t \frac{W_t^i}{W_t^{m,P}} dt \tag{6}$$

where π_1^P, π_2^P denote the percentage investments in the risky assets in the manager's personal account. The first term of the right hand side of Equation (6) accounts for investments in the riskless asset; the second and third terms account for changes in the values of the risky assets; the term $\gamma \phi_t \frac{W_t^i}{W_t^{m,P}}$ accounts for the fees paid by the investor to the manager per unit of time.

We assume that, in her personal account, the manager faces the same margin requirements as in Equation (3), and that she cannot take a position in her personal account that would conflict with a position taken by the fund; i.e.,

$$\pi_i^F \pi_i^P \geq 0, i = 1, 2$$

Case B: Restricted Manager

If the manager is not allowed to access the risky assets in her personal account, the dynamics of her wealth in her personal account, $W^{m,P}$ are given by

$$\frac{dW_t^{m,P}}{W_t^{m,P}} = \frac{dS_{0,t}}{S_{0,t}} + \gamma\phi_t \frac{W_t^i}{W_t^{m,P}} dt$$

Optimization problem

We model the interaction between the investor and the fund manager as a Stackelberg differential game, with the manager being the leader. In this game, at each time t, the manager determines the asset allocations of the fund and then the investor decides the amount to invest in the fund. Given the fund allocations, the investor maximizes his expected utility from terminal wealth. We assume that the time horizon of the investor is T, and that the investor has logarithmic preferences. Given logarithmic preferences, it is possible to calculate the investor's allocation to the fund, ϕ, by solving the myopic utility maximization problem.[15] The optimal investment to the fund is given by the ratio of the expected excess return of the net-of-fees investment in the fund, over the variance of the same.

$$\phi_t^* = [\vartheta_t^{\pi_1^F,\pi_2^F}]^+ = \max\left(\frac{(\pi_{1,t}^F\sigma_{11} + \pi_{2,t}^F\sigma_{21})\xi_1 + \pi_{2,t}^F\sigma_{22}\xi_{2,t} - \gamma}{(\pi_{1,t}^F\sigma_{11} + \pi_{2,t}^F\sigma_{21})^2 + (\sigma_{22}\pi_{2,t}^F)^2}, 0\right) \tag{7}$$

The manager faces the following optimization problem: she needs to choose the allocation of the fund to the risky assets, as well as the allocation to the risky assets in her personal account. Her objective is to maximize her expected utility of terminal wealth. We assume that she has the same time horizon as the investor and constant relative risk aversion preferences with risk aversion coefficient α. In contrast to the investor, the manager is non-myopic and strategic; in determining her portfolio she accounts for potential future changes in the

[15] Myopic investor preferences are crucial in the tractability and in our ability to solve the problem. Our choice of logarithmic preferences follows Hugonnier and Kaniel (2010). An alternative would be to assume an overlapping generation structure for investors, similar to He and Krishnamurthy (2013).

opportunity set, as well as on how her allocation decisions impact current and future investor flows.

The value function of the manager is given by

$$V(t,x,y,p,\xi_2) = \max_{(\pi_1^F,\pi_2^F,\pi_1^P,\pi_2^P)\in\Pi} \mathbb{E}\left[\frac{(W_T^m)^{1-\alpha}}{1-\alpha} \middle| W_t^m = x, W_t^i = y, p_t = p, \xi_{2,t} = \xi_2\right] \quad (8)$$

where Π is the set of allowed strategies for the asset allocations.[16]

Lemma 3.1. *Given the preferences of the manager, and the assumptions outlined above, the manager's value function is homogeneous of degree* $1-\alpha$ *in the manager's and the investor's wealth.*

We provide a proof of Lemma 3.1 in the Appendix. This lemma implies that the manager's optimization problem can be described in terms of the ratio of the manager's wealth to the investor's wealth resulting in one fewer state variable in the optimization problem described in Equation (8). The state variables necessary to solve the problem are: i) the ratio of the wealth of the manager to the wealth of the investor; ii) the level of the mean-reverting process corresponding to a component of the manager's ability; and, iii) the percentage of the manager's wealth committed to the fund. The choice variables are the allocations in the risky assets both inside the fund and in the manager's personal account. In the most general case there are 3 state variables — described above— and 4 choice variables — corresponding to fund and manager personal allocations in each of two risky assets — for each time period.

4. Results

To solve the model described in Section 3, we use an algorithm based on dynamic programming. While in the case of a manager that does not commit her wealth to the fund and who

[16] When the manager is restricted from using her investment ability in her personal account $\pi_1^p = \pi_2^p = 0$.

is allowed to take positions in her personal account that may conflict with the positions of the fund it is possible to obtain a closed form representation of the solutions, given the additional complexity arising from the constraints we impose on the manager, we provide numerical solutions instead.[17] The details of the numerical method are described in Appendix B.

Overall, our results can be understood in terms of the conflict of two incentives for the manager: her desire to choose a fund allocation that generates large fees from the investor; and her concern for hedging, both in terms of diversification and intertemporal demands. In the absence of this conflict, for example if the manager were able to hedge her implicit exposure to the risky investment opportunity generated from the fees she receives from the investor through trading in her personal account, she would choose the fund allocation that extracts the largest net present value of fees possible from the investor. All else equal, a higher (lower) fee rate leads the investor to allocate less (more) capital to the fund. To maintain the investor's effective exposure to equity and the fees received constant, the manager can de-lever (lever) the fund portfolio accordingly, implying a positive relation between the fee rate and the fund expected return and volatility. Furthermore, since the optimal effective investor exposure to equity, in terms of rents that can be extracted by the manager, is independent from the managers ability the funds allocation to a risky asset will decrease in ability. However, since in our framework the manager faces restrictions in her personal account; e.g., margin requirements, she also needs to consider her own diversification and intertemporal hedging demands.

[17]The setting where the manager can take positions in her personal account that may conflict with those of the fund is discussed in Hugonnier and Kaniel (2010). They show that the manager uses her personal account to hedge the implicit exposure to the fees received from the fund, while choosing the fund allocations to balance maximizing fees collected from the investor, and intertemporal hedging of the fees. In the case of a constant opportunity set, Hugonnier and Kaniel (2010) show that the optimal fund allocation corresponds to the allocation that maximizes immediate investor participation. The same is true for a stochastic opportunity set of a single risky asset. When the investment opportunity set includes multiple assets whose expected return varies stochastically, the optimal fund allocation is provided in terms of the solution to a backward stochastic differential equation.

Benchmark: Maximizing Investor Participation

A useful benchmark for fund allocations is the fund allocation that maximizes investor participation. Deviations from the allocation that maximizes investor participation serve as a measure of the impact of the constraints placed on the manager.

Lemma 4.1. *Assuming that the fund fee rate, γ, is positive, the fund allocation that maximizes investor participation is given by*

$$\pi_{1,t}^{F,*} = \frac{2\gamma\xi_1}{\sigma_{11}\left(\xi_1^2+\xi_{2,t}^2\right)} - \frac{2\gamma\xi_{2,t}}{\sigma_{22}\left(\xi_1^2+\xi_{2,t}^2\right)}\frac{\sigma_{21}}{\sigma_{11}}$$
$$\pi_{2,t}^{F,*} = \frac{2\gamma\xi_{2,t}}{\sigma_{22}\left(\xi_1^2+\xi_{2,t}^2\right)}$$

For this fund allocation, the expected excess return, idiosyncratic volatility, and Sharpe ratio of the fund are given by:

$$\textit{Fund Expected excess return} = \gamma$$
$$\textit{Fund Idiosyncratic volatility} = \frac{2\gamma}{\sqrt{\xi_1^2+\xi_{2,t}^2}}$$
$$\textit{Fund Sharpe ratio} = \frac{\sqrt{\xi_1^2+\xi_{2,t}^2}}{2}$$

Lemma 4.1 can be directly proven by first calculating the allocation of the investor, given an allocation for the fund, and then solving the first order conditions. Lemma 4.1 indicates that differences in managerial ability are expressed through the idiosyncratic volatility of fund returns. If the fee rate of two separate funds is the same, the expected excess return for both funds would be the same, but the idiosyncratic volatility will be lower for the returns of the fund managed by the manager of higher ability. Empirically, Lemma 4.1 suggests that if one wants to capture managerial ability one should focus on the levels of idiosyncratic volatility rather than expected fund returns. Regarding fund allocations in the risky assets, they are proportional to the fee charged by the fund, and inversely proportional to managerial ability.

A. Base Case: Constant Opportunity Set

For our base case, we consider a manager whose ability is in terms of the fund having access to a risky asset with constant Sharpe ratio. The values of the model parameters are the ones provided in Hugonnier and Kaniel (2010). These parameters are based on Wachter (2002), Barberis (2000), and Campbell and Viceira (1999). The margin requirements are set to the maintenance margin requirements specified in Regulation T which are 50% for a long position and 150% for a short position, see Fortune (2000). The parameter values are provided in Table 1.

A.1. Varying ability

Lemma 4.1 indicates that in the case of a manager that maximizes investor participation, the manager chooses an allocation that achieves a constant expected excess return, equal to the fee rate. In that case, the idiosyncratic volatility of fund returns are inversely proportional, and the Sharpe ratio is proportional, to the ability of the manager.

In Figure 1 we explore the impact that restrictions placed on the manager have on these quantities. We consider the case of an unrestricted manager that does not commit any of her wealth in the fund and who can access the risky assets available to the fund in her personal account; the case of a restricted manager that does not commit any of her wealth to the fund and who cannot access the risky assets available to the fund in her personal account; and the case of a manager that commits all her wealth to the fund.

The fund expected excess return, idiosyncratic volatility, and Sharpe ratio, presented in Figure 1 indicate that the manager's choices deviate from the choices of a manager that tries to maximize investor participation at low levels of managerial ability for a manager whose wealth is fully committed to the fund. In this case, due to the low ability of the manager, the size of the fund is relatively small, and managerial preferences are relatively more important. The effect is only significant for a manager that commits her entire wealth to the fund, an indication that the other types of managers are able to adjust their personal accounts in response to the smaller fund size. [18]

A.2. Wealth equivalents

While expected excess returns, levels of idiosyncratic volatility, and Sharpe ratios for different types of managers can be directly mapped to observed quantities, there are additional measures that can be used to quantify the cost of imposing restrictions on fund managers. One such measure is the wealth equivalent for the manager, and the wealth equivalent for the investor. In each case, the wealth equivalent is equal to the additional wealth that the manager (or the investor), would require in order to be indifferent between accepting a restriction – with the additional wealth – or not accepting this restriction. Since the investor has logarithmic preferences, his wealth equivalent is given by

$$\ln\left((1+\mathrm{weq}_{\mathrm{inv}})\mathrm{Utility\ with\ restriction}\right) = \ln(\mathrm{Utility\ without\ restriction})$$

while for the case of manager, with constant relative risk aversion, the wealth equivalent for the manager is given by

$$\left((1+\mathrm{weq}_{\mathrm{man}})^{1-\alpha}\mathrm{Utility\ with\ restriction}\right) = \mathrm{Utility\ without\ restriction}$$

[18] In results we do not report, we have found that margin requirements and trading constraints in the unrestricted manager's personal account are binding, either at their minimum value – zero – or their maximum value – twice the manager's wealth – in most cases. The cases where the margin requirements and trading constraints are not binding for an unrestricted manager are easy to determine from the plots: they arise when the fund allocation in the risky asset maximizes investor participation.

We consider the case of an unrestricted manager as the base with respect to which other cases are measured. It is clear that the utility of managers is reduced if restrictions are placed on them, either in terms of their ability to access the risky asset in their personal account – the case of a restricted manager – or in terms of committing their personal wealth to the fund. Intuitively, since unrestricted managers are better able to manage their implicit exposure to the risky asset through the fees they receive from the fund, investors facing a restricted manager or a manager whose personal wealth is fully committed to the fund they manage, are likely to be better off compared to investors facing a manager that does not commit her wealth to the fund and is able to access the risky asset in her personal account.

Table 2 presents the wealth equivalents for the basecase of managers with constant levels of ability. From the table we note that the wealth equivalent for the investors is minimal between the different restrictions facing managers. On the other hand, for managers, the wealth equivalent loss may be significant. While, for managers restricted from accessing the risky asset in their personal account, the cost of the restriction is monotonically increasing as the Sharpe ratio of the risky asset improves, the cost is non-monotone for a fully committed manager. For small values of the Sharpe ratio the increased cost is due to having to fully commit to allocations to the risky asset that are higher than the manager would prefer, in order to attract the investor. The cost decreases as the risky asset becomes more attractive, but increases again for high levels of the Sharpe ratio, because, to increase investor participation, the fund limits the investment in the risky asset, and the manager is not able to use a personal account to compensate.

A.3. Varying Commitment Level

For the parameter values in Table 1, the top panel of Figure 3 displays the fund allocations for different commitment levels. In Figure 3 we plot the allocations for two different managers, one with wealth equal to 1% of the wealth of the investor, and another with wealth equal to 10% of the wealth of the investor. For a manager that is not restricted from holding the risky asset in her personal account, the top panel of the figure illustrates that increasing the level of commitment leads to a decline in fund allocations to the risky asset, away from the level that generates the largest fees (which in this case is 73%), and toward the level suggested by the manager's personal preferences (which in this case is 60%). The result is more pronounced for the manager whose wealth is larger relative to the size of the fund.

For a manager that is restricted from accessing the risky asset in her personal account, the bottom panel of Figure 3 illustrates the dependence of the fund allocations on the manager's commitment level. The figure reveals that a manager with relatively little wealth chooses an allocation level close to the level that maximizes investor participation. On the other hand, for a relatively wealthy manager the effect of commitment is not monotonic: when the commitment level is 0% the fund allocation to the risky asset is higher than the level that maximizes investor participation since the manager can only implicitly access the risky asset through the fees she receives. As the commitment level increases above 0% the manager's exposure to the risky asset is both explicit, through her investment in the fund, and implicit, through the fees she receives. In this case, perhaps surprisingly, the manager chooses a higher fund allocation in the risky asset than in the case of zero commitment. Intuitively, this increase is based on the interplay between the two types of exposure the manager has to the risky asset. Increasing fund allocation in the risky asset increases the expected return of the fund and contributes to higher expected returns for the portion of the manager's wealth directly invested in the fund, but, on the other hand, decreases investor participation and consequently the fees the manager receives. As a result, and in combination with the manager's diversification motives, the optimal fund allocation in the risky asset increases for positive, but small, commitment levels. For larger levels of commitment, closer to the case where the manager commits 100% of her wealth to the fund, a fund allocation at the level that would maximize investor participation results in the manager having too large an exposure to the risky asset — both through direct investment and through the fees the manager receives. This large exposure leads the manager to choose a lower level of fund allocation in the risky asset. This non-monotonic behavior is evident in the case of a relatively wealthy manager, whereas for the manager with relatively little wealth, wealth concerns dominate, and the manager chooses a fund allocation that is closer to the fund allocation that maximizes investor participation for all commitment levels.

B. Stochastic, Mean-Reverting, Ability

Managers with stochastic, mean-reverting, levels of ability are modeled by an opportunity set that includes a risky asset with a mean-reverting level of Sharpe ratio which follows a CIR process. We choose the parameters of our model to match the expected time that the process takes to revert to half a standard deviation above its long term level when starting one standard deviation above its long term level to the same expected time as in the model by Hugonnier and Kaniel (2010).[19] The remaining parameters are the same as in the case of the manager with constant ability. The parameters are presented in Table 3.

Figure 2 presents the expected excess return, idiosyncratic volatility, and Sharpe ratio for a fund managed by a manager with stochastic, mean-reverting, ability. Differences between the choices of the fund manager and the choices of a manager that maximizes investor participation are more pronounced, relative to the case of a manager with constant ability: for example, when managerial ability is temporarily low, we observe large deviations between a manager that commits her entire wealth to the fund and the managers with a personal account. These deviations are due to the margin constraints on the manager's personal account allocations and the margin constraint on the fund. Interestingly, leveraging occurs mostly in the manager's personal account. Since the manager that commits her entire wealth to the fund does not have a personal account, she instead leverages the fund allocations.

The non-monotonicity observed in Figure 2 for the expected return for the fully committed manager, is due to the fact that, for very low, temporary, levels of the manager's ability, the margin constraint on the fund allocation to the risky asset binds, restricting the implicit expo-

[19]Hugonnier and Kaniel (2010) choose an Ornstein-Uhlenbeck process, rather than a CIR one to represent time-varying ability.

sure of the manager to the risky asset. With no margin constraints imposed on the fund trading strategy, for a fully committed manager expected returns decline as current ability increases.

Figure 4 presents fund allocations for managers with mean-reverting ability for different levels of commitment. The top panel presents results for a manager that can access the risky asset in her personal account, while the bottom panel focuses in the case where the manager is restricted from accessing the risky asset in her personal account. Both panels present results for managers whose wealth is 1% and 10% of the wealth of the investor. The unconditional mean level of ability is set at the same level as in Figure 3, in the case where the manager's ability is constant. Furthermore, the initial, temporary, level of ability is also set to this value. The fund allocation to the risky asset that maximizes investor participation is 73%, while, in the absence of the investor, the manager would like to hold 70% of her wealth in the risky asset.

Both panels of Figure 4 indicate that the fund allocation chosen by the manager stays close to the level that maximizes investor participation. We find that the allocation to the risky asset increases as the commitment level increases, with the increase being greater for the relatively wealthy manager. This result is contrary to the result reported in Hugonnier and Kaniel (2010), who find that managers that are free to hedge their exposure to the fees they receive, choose fund allocations below those that maximize investor participation. The Hugonnier and Kaniel (2010) result is driven by the manager's intertemporal flow hedging motives: the manager distorts the fund portfolio resulting in temporarily extracting less fees from the investor when her ability is temporarily high so that she can extract more, from a wealthier investor, when the manager's ability is temporarily lower. The reason for the discrepancy is the margin requirement that the manager faces in her personal account in our framework: the unrestricted manager in Figure 4 chooses the maximum allocation to the risky asset in her personal account for all commitment levels.[20] This binding constraint in the

[20]The manager's investment choice in her personal account can be understood by her personal intertemporal hedging demand: in the absence of the investor, a manager that faces a constant opportunity set chooses a 60% allocation to the risky asset. When the opportunity set has a mean-reverting Sharpe ratio, her optimal allocation

allocation of the manager's personal account leads her to increase the fund allocation to the risky asset as her commitment to the fund increases. The same intuition explains the choices of the manager that is restricted from accessing the risky asset in her personal account.

We point out that the constraints lead to higher fund expected returns and higher idiosyncratic volatilities for managers that commit their entire wealth to the fund, compared to either managers restricted from investing in the risky asset in their personal account, or managers who are able to access the risky asset in their personal account. Sharpe ratios are higher for fully committed managers. While these results are valid for the parameter range we have chosen, they may not hold for all cases.

To analyze the interplay between constant and stochastic ability, we also consider the case of a manager that combines a constant level of ability, and an additional, stochastic, mean-reverting level of ability. This manager is modeled by allowing the opportunity set of the fund to consist of two risky assets: one with a constant Sharpe ratio and one whose Sharpe ratio has a mean-reverting component. The results, for the case of zero correlation between the assets, are presented in Figure 5. The results are in line with the intuition developed in the case of a manager with constant ability and a manager with stochastic, mean-reverting, ability. The biggest difference is that, due to the diversification between the two assets, the fund investment in the risky assets is smaller than in the case of only having access to a single risky asset. The fund allocation that maximizes investor participation is 36.4% in each of the assets, when the temporary level of the Sharpe ratio for the asset with a mean-reverting Sharpe ratio is at its unconditional mean, which is assumed to be the same as the level of the Sharpe ratio of the risky asset with a constant Sharpe ratio. In the absence of the investor, the optimal personal portfolio of the manager consists of a 60% investment in the asset with constant Sharpe ratio and an additional 70% investment in the asset with a Sharpe ratio with a mean-reverting component.

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to the risky asset rises to 70%, indicating positive intertemporal hedging demand. This demand is magnified in the presence of the investor since the manager's wealth is implicitly higher, due to the expected fees she receives.

Table 4 presents the optimal asset allocations as the correlation between the two assets varies between -50% and 50%, and the level of the mean-reverting component of the Sharpe ratio of the second risky asset varies between one-half standard deviation below and one-half standard deviation above the long-term level. The allocations stay close to the values that maximize investor participation for all manager types. Similar to the case of zero correlation, the hedging demand is positive for the MR asset, and is greatest for the fully committed manager.

From the table, we notice that the changes in the allocation are concentrated in the asset with constant Sharpe ratio (GBM), while the allocation in the second asset (MR) remains relatively stable. The intuition behind this result center on the intertemporal hedging demands for the two different assets: while the mean-reverting asset generates intertemporal hedging demands, with the manager choosing an allocation that avoids extracting the maximum amount from an investor when expected returns are high, in order for the investor's wealth to grow and to be able to extract more when expected returns are low, the asset with the constant Sharpe ratio does not generate intertemporal hedging demands. This difference between the assets implies that, as the correlation between their returns changes, it is more efficient to rebalance the asset with constant Sharpe ratio to achieve the desired diversification and overall equity exposure, rather than the asset with the mean-reverting Sharpe ratio.

C. Comparative Statics: Fees, Risk Aversion

Figures 6 and 7 present fund allocations for a fund managed by a manager with constant level of ability as the fee rate paid to the manager and the manager's risk aversion vary. The three types of managers correspond to a manager that does not commit her wealth to the fund and can invest directly in the risky asset in her personal account (unrestricted); a manager that does not invest her wealth in the fund but is restricted from investing in the risky asset in her personal account (restricted); and a manager that commits her entire wealth to the fund (fully committed).

Figure 6 shows that fund allocations to the risky asset increase as the fund's fee rate increases, and that fund allocations are close to the allocations that maximize investor participation. The only deviation occurs for the fully committed manager at high fee rates, where the manager chooses an allocation that is slightly lower than the choice that maximizes investor participation due to her preferences and her inability to hedge exposure to the risky asset through a personal account. We note that the fund allocations are discontinuous when fees are equal to zero: in that case the manager cannot benefit from investment in the fund and chooses fund allocations based on her own preferences.[21] When managers are able to choose allocations that maximize investor participation, then the level of fees does not affect the investor's utility – a result that is in line with the results discussed in Hugonnier and Kaniel (2010). We have verified that even for fully committed managers, the change in the utility of the investor is minimal as the level of the fees changes.

Figure 7 shows that fund allocations decrease as the manager's risk aversion increases. It is interesting to note that there exists a range of risk aversions — approximately 1.5 to 3 — where the unrestricted manager chooses the fund allocation that maximizes investor participation. In this situation the manager is able to hedge her implicit exposure to the risky asset from the fees she receives through changes in the allocations in her personal account. Deviations only occur when one of the constraints placed on the manager binds (either the margin requirement or the no-shorting constraint). We note that even in the case when managerial risk aversion matches investor risk aversion; i.e, when both have logarithmic preferences, fund allocations do not change significantly.[22]

[21] In the case of managers who do not commit any of their wealth to the fund, the fund allocations are not well defined.

[22] For the level of ability we consider, an investor with logarithmic preferences would choose an optimal allocation of 180% of wealth in the risky asset.

D. Learning Managerial Ability

In addition to considering the effect of manager commitment, we can also extend our framework to address the dynamics of asset allocation when the ability of the fund manager is unknown. In that situation, the manager, as well as the investor, share a common belief (prior) on the manager's ability, and continuously update that belief based on the fund's performance. Our approach is an extension of the model described by Dangl, Wu, and Zechner (2008). Similar to Dangl, Wu, and Zechner (2008), the fund manager is able to choose the fund's volatility and expected return. In Dangl, Wu, and Zechner (2008), this choice is made in reduced form, while in our model, we explicitly account for the portfolio composition by allowing the manager to choose the investment of the fund in the risky asset. Unlike Dangl, Wu, and Zechner (2008) we do not assume that fund returns are decreasing with fund size, as such an assumption would introduce an additional state variable — fund size — to the problem. Instead, we focus on the portfolio composition of the fund, and the dynamics induced by learning.

We assume that the information is symmetric between the manager and the investor; i.e., the level of ability of the manager is unknown to both; they both share a common prior on the value of the manager's ability; the investor is able to observe the fund asset allocation to the risky asset; and both the manager and investor updated their beliefs once returns are observed. The setup of the problem is described in detail in Appendix C.

We have implemented this model and found that the allocations in the fund and manager's personal account are largely identical with the ones found when managerial ability is known, so long as the level of the known managerial ability equals the mean of the distribution of the unknown ability. We do note however that there are differences between the known and unknown ability levels: since the level of ability is updated following either positive or negative fund returns, in the case of unknown ability we expect that the fund volatility will change over time. For example, following positive returns, investors and manager update their beliefs on the manager's ability upward, and the manager changes the fund allocation to reduce volatility; following negative returns the update is negative and fund volatility increases. In addition, the flows into and out of the fund resemble returns: after positive (negative) returns, the investor and manager update managerial ability upward (downward), leading the manager to reduce (increase) fund volatility and the investor to increase (decrease) his investment.

5. Competition between Managers

We have so far assumed that there is only one fund available to the investor. In this section we introduce competition between fund managers. Competition has the potential to alleviate the agency problem between the managers and the investor: when the investor only has access to a single fund, the fund manager has the incentive to choose an asset allocation that extracts large fees from the investor. Allowing the investor to choose between several funds induces competition between the funds. If the funds have access to identical opportunity sets competition would result in the funds aligning themselves with the investor's objectives in an effort to attract the investor.

To keep notation tractable, we restrict our attention to the case of two funds, but our results and intuition extend to the case of multiple funds and assets. In additional to the funds, the opportunity set of the investor includes access to a money market account and the market portfolio. The opportunity set of each fund includes the money market account and a risky asset whose returns are orthogonal to the returns of the market portfolio but may be correlated with each other.[23]

Similar to the single fund case, we assume that the interaction between the fund managers and the investor follows a Stackelberg differential game, with the fund managers being the leaders. The investor is assumed to have logarithmic preferences and maximizes expected utility from terminal wealth. The detailed description of the setup is provided in Appendix D.

[23]Masmoudi (2006) considers the case where the two managers have access to identical opportunity sets, and studies equilibria that are Pareto optimal, although not necessarily Nash.

We are able to find the fund asset allocations that maximize investor participation in closed form. Denoting by $\sigma_i, \xi_i, i = 1,2$, the volatility, and Sharpe ratio, for the risky asset for each fund, ρ the correlation of the returns of the risky assets, $\gamma_i, i = 1,2$, the proportional fees charged by each fund, and $\pi_i, i = 1,2$, the allocations the following lemma characterizes the fund allocations, size, expected excess return, idiosyncratic volatilility, and Sharpe ratios.

Lemma 5.1. *The fund allocations that maximize investor participation are given by*

$$\pi_1 = \frac{(4-\rho^2)\gamma_1}{\sigma_1\left(2\xi_1 - \rho^2\xi_1 - \rho\xi_2\right)}$$
$$\pi_2 = \frac{(4-\rho^2)\gamma_2}{\sigma_2\left(2\xi_2 - \rho^2\xi_2 - \rho\xi_1\right)}$$

For this fund allocation, the fund size, expected excess return, idiosyncratic volatility, and Sharpe ratio for each fund are given by:

$$\begin{aligned}
\textit{Fund size}_i &= \frac{\left((2-\rho^2)\xi_i - \rho\xi_j\right)^2}{\gamma_i(4-\rho^2)^2(1-\rho^2)}, \quad i,j = 1,2, j \neq i \\
\textit{Expected excess return}_i &= \gamma_i \frac{2\xi_i - \rho\xi_j}{(2-\rho^2)\xi_i - \rho\xi_j}, \quad i,j = 1,2, j \neq i \\
\textit{Idiosyncratic volatility}_i &= \frac{(4-\rho^2)\gamma_i}{(2-\rho^2)\xi_i - \rho\xi_j}, \quad i,j = 1,2, j \neq i \\
\textit{Sharpe ratio}_i &= \frac{2\xi_i - \rho\xi_j}{4-\rho^2}, \quad i,j = 1,2, j \neq i
\end{aligned}$$

The fund allocations provide insight on the robustness of the results in the case with a single manager in the following corrolaries for the case when managers charge the same fees, $\gamma_1 = \gamma_2$, and who have ability with non-negative, $\rho \geq 0$, and whose ability satisfies $\xi_2 \leq \xi_1 \leq \xi_2(2-\rho^2)/\rho$.

Corollary 5.2. *In the case when correlation of returns is equal to zero,* $\rho = 0$, *the fund allocations are the same as in the single manager case.*

Corollary 5.3. *The manager with higher ability manages a larger fund*

$$\phi_1 \geq \phi_2$$

where equality only holds when the ability of the two managers is the same, $\xi_1 = \xi_2$.

Corollary 5.4. *The idiosyncratic volatility of a fund decreases as the ability of the fund manager increases, while the idiosyncratic volatility increases as the ability of the other fund manager increases; the idiosyncratic volatility of each fund increases as the correlation between the ability of the fund managers increases.*

Corollary 5.5. *The Sharpe ratio of the returns of the fund managed by the manager with higher ability is higher than the Sharpe ratio of the returns of the fund managed by the manager with lower ability; the Sharpe ratio of a fund increases as the ability of the fund manager increases, while the Sharpe ratio decreases as the ability of the other fund manager increases.*

These results indicate that the intuition developed in the case of a single manager carries over to the case of competition between managers, as long as managers have ability that is not perfectly correlated with each other. In the case of a single manager, and in the absence of competition, we have shown that managers adjust fund allocations to maximize investor participation by limiting the investor's access to the manager's ability. With competing managers fund allocations depend on the correlation between the ability of the managers, but the intuition remains the same: if ability is uncorrelated, then each manager acts as a monopolist; increasing correlation leads to increasing competition, and the managers make more of their ability available through the fund, increasing the idiosyncratic volatility of fund returns. When ability is different among managers, the manager with the lower ability level competes with the manager with higher ability by making more of her idiosyncratic ability available to the investor, leading to an increase in the idiosyncratic volatility of the fund. Overall, the results suggest that the intuition from the single manager case, that ability is mainly reflected through increased Sharpe ratios but not necessarily increased expected returns, survives in the case of multiple, competing, managers, as long as idiosyncratic returns are not perfectly correlated.[24]

A. Numerical Results: Competing Managers

While the theoretical results on maximizing investor participation with competing funds are a useful guide, they do not consider the intertemporal concerns of managers. In this section we numerically explore the impact of competition among funds. We limit ourselves to the case of two competing funds and consider competition between managers with constant levels of ability as well as between a manager with constant level of ability and a manager with a mean-reverting level of ability. Since we interpret the manager's ability as the generation of excess returns by taking idiosyncratic risk, we consider cases where the managers ability is correlated and allow the managers to either have the same ability level; or one manager to have a higher ability level than the other. We also assume that both managers commit 100% of their wealth to their fund, and show that the impact of this assumption is relatively minor when the ratio of the managers' wealth to the wealth of the investor is small.[25]

A.1. Competing Managers with the Same Level of Constant Ability

We first consider the case of two managers with the same level of constant ability, represented in our framework by the funds having access to one risky asset each, that follows geometric Brownian motion with a constant Sharpe ratio level. Since, the intuition from the theoretical results for multiple managers with uncorrelated ability that maximize investor participation, is

[24]We also note that the ability of the investor to access a risky asset – the market portfolio – does not impact the fund strategies. In our setup, this is due to the independence between the returns of the market portfolio and the returns of the risky assets available to the managers. However, even in the case where the returns of the assets available to the funds were imperfectly correlated with the returns of the risky assets available to the managers, the result is qualitatively similar: the managers compete in the part of their ability that is reflected in returns orthogonal to the returns of the market portfolio.

[25]Unless mentioned, the values for all parameters for both funds are equal, and set to the values in Tables 1 and 3.

that managers optimally follow monopolistic strategies, we consider only the case of managers with correlated ability.

In Figure 8, we present the fund allocations when the correlation between the managers' ability is 50%. Given the symmetry in the problem, we focus on the symmetric Nash equilibrium and present the strategy of only one of the fund managers. The top panel presents the fund's allocation to the risky asset, while the bottom panel presents the difference between the allocation chosen by a monopolistic fund and the allocation chosen by the competing fund.

From the figures, we conclude that competition leads to significant increases in the fund allocations. While a monopolistic fund would choose an allocation close to 73%, which would extract the largest investment from the investor, competing funds try to attract the investor by choosing allocations that are up to 40% higher. Competition is most fierce when the wealth of the managers is small. When the wealth of one of the managers is large relative to the wealth of the other manager, the relatively poorer manager tilts her portfolio more towards the riskier asset and attracts most of the investment.

B. Competing Managers with Different Levels of Constant Ability

To determine the impact of competition on managers with different levels of constant ability, in Figure 9 we study the case where the two managers have access to correlated assets with correlation 50%, but where the Sharpe ratio of the asset available to one manager is lower than the Sharpe ratio of the asset available to the other manager. When the ratio of the managers' wealth to the wealth of the investor is small, the managers compete by increasing the fund allocations to the risky asset. The allocation increase is larger for the case of the manager with the smaller level of ability, since in order to attract the investor she needs to increase the expected return of the fund, leading to higher idiosyncratic volatility for fund returns.

A discontinuity in the fund allocations occurs when the ratio of the wealth of the manager with the lower level of ability to the wealth of the investor increases. Intuitively, in this case, the manager with the lower level of ability cannot choose an allocation that is attractive to the investor, and she drops out of the competition by choosing an allocation that satisfies her own preferences. In this case the manager with the higher level of ability becomes a monopolist and chooses the fund allocation in the risky asset to be approximately 73%, which achieves the highest participation from the investor.

C. Manager with Constant Level of Ability vs. Manager with Mean-Reverting Level of Ability

In Figure 10, we study the fund allocations for the case of two competing managers, one of which has a constant level of ability while the other has a mean-reverting level of ability. In the figure, the long-term and current ability levels of the manager with the mean-reverting ability is chosen to be equal to the ability level of the manager with the constant level of ability. As in the case of competing managers with constant ability levels, we notice that competition leads both funds to increase their allocation to the risky assets.

The non-monotonic behavior, observed in the figure, for the manager with mean reverting ability is driven by flow hedging demand at low levels of the ratio of the manager's wealth to the investor's wealth, which tends to depress the allocation in the risky asset in the fund. As fund size increases, there is less flow hedging demand and the allocation to the risky asset increases to a level higher than what the manager would hold on her own. When the manager becomes relatively wealthy, compared to the investor, the impact of the decline in flow hedging demands is mitigated and the manager's own preferences dominate, leading to a decrease in the fund allocation in the risky asset. Competition between managers transmits the non-monotonicity in the holdings of the fund with the manager with mean reverting ability to non-monotonic holdings for the fund with a with constant ability manager as well.[26]

[26] In unreported results, we find a similar non-monotonic behavior for the case of a single manager with mean-reverting ability but not for a single manager with constant ability.

C.1. 0% Commitment vs. 100% Commitment

To determine the impact of commitment of the managers' wealth on the fund allocations, in Table 5 we present the difference in allocations between the case where managers do not commit any of their wealth to the fund but are not allowed to invest in the same opportunity set as the fund in their personal accounts, i.e., they are restricted to opportunities available to the retail investor vs. the case where the managers commit 100% of their wealth to the fund. In our framework this corresponds to restricting the managers from investing in the risky asset in their personal accounts. From the table we see that the differences in the fund allocations are relatively small, particularly when the ratio of wealth of the managers to the wealth of the investor is small. Other than the case of competing managers with one manager have a mean-reverting level of ability, restricting the managers from investing in the same risky assets as the funds leads to slightly larger fund allocations in the risky asset. The results are more complicated in the case of competition between a manager with mean-reverting ability and a manager with constant ability. The mean-reversion in the ability of one of the managers increases her intertemporal hedging demand, especially when there are restrictions to the manager's personal portfolio, resulting to reduced competition for the other manager.

Overall, the results of Table 5 justify focusing our study of competing managers to the case of managers that commit 100% of their wealth to their funds.

6. Conclusions

We have presented a model where fund managers have the ability to generate excess returns by taking idiosyncratic risk and studied how forcing the managers to commit part of their personal wealth to the fund they manage affects fund allocations. We find that fund expected returns are largely independent of ability, while idiosyncratic volatility decreases as managerial ability increases. Placing restrictions on the choices that managers can make in their personal accounts has a relatively small effect on fund allocations for managers of constant ability. On the other hand, managers whose ability varies over time do change fund allocations as a function of restrictions placed on their behavior. We found that these changes result in higher expected returns, and higher idiosyncratic volatilities.

In addition to our results on fund expected returns and idiosyncratic volatility, we have shown that, as a result of the interactions between the manager's objective and restrictions, and the investor's objective, the fund allocation in the asset that represents managerial ability can vary non-monotonically with respect to the ratio of manager to investor wealth, and the commitment of the manager's wealth to the fund. To quantify the impact of managerial commitment on the manager and the investor we calculated wealth equivalents and showed that, while for the parameters we consider the effect of managerial commitment on the investor is minor, commitment makes a difference for the manager. This impact on manager wealth equivalent, varies non-monotonically with manager ability.

Our results also suggest that managers with higher ability manage larger funds, without necessarily achieving higher returns. This result is similar to the results in Berk and Green (2004) but in a very different setting. While Berk and Green (2004) assume that expected returns, and managerial ability, is decreasing in the size of the fund, we allow constant returns to scale, and derive our results endogenously, by allowing managers to adjust the access investors have to managerial ability through the fund in a way that varies with the level of ability.

We were able to demonstrate the robustness of our results in the case where managers with related ability compete, and characterize the extent of competition by the correlation between the returns of the assets that represent managerial ability. Managers with uncorrelated ability are effectively monopolists that are able to extract rents from the investors. While competition does influence fund allocations when ability is correlated across managers, unless correlation is perfect, managers are still able to extract some rents from the investors. One effect of competition is that non-monotonic holdings in the risky asset in the fund may arise due to changes in the ability of the other manager.

Our results have potential testable empirical implications. For example, one way to test the result that managers with different levels of ability chose fund allocations that differ in their idiosyncratic volatilities, would be to look at changes at idiosyncratic volatilities in fund returns around manager changes: following a departure of a manager we expect the level of idiosyncratic volatility of fund returns to change. If fund returns are above average before the departure of the previous manager, we would expect that the next manager would have, a priori, a lower level of ability, and increase idiosyncratic volatility. Similarly, if fund returns are below average before the departure of the previous manager, we would expect that the next manager would have, a priori, a higher level of ability, and decrease idiosyncratic volatility. Similar empirical predictions hold for funds with different levels of fees. Based on our results on learning, we would also expect that a manager would increase the idiosyncratic volatility of returns following negative returns, and decrease it following positive returns. Our results regarding competing fund managers indicate that correlation of managerial ability, which can be measured through the correlation of fund returns, is an important determinant of fund expected returns and fund return volatility. This relationship can be investigated by computing the correlation of fund returns with a fund index and determining whether funds with low correlation to the index have higher or lower idiosyncratic volatility.

We have been able to show that our results extend in the case where managers with related ability compete, but where knowledge of managerial ability is symmetric between managers and investors. An interesting further extension would be to consider a case of competing managers with asymmetric information between the manager and the investors; i.e., when the manager knows more about her own ability compared to what the investors know. Such an extension would capture managerial career concerns in the optimal fund allocation choice, but would be significantly harder to solve. We leave the extension for future work.

Appendix A. Homogeneity of the Value Function

In this Appendix we provide a proof for the homogeneity of the value function defined by Equation (8) in the variables of the manager's and the investor's wealth, i.e.,

$$V(t, cx, cy, \rho, \xi) = c^{1-\alpha} V(t, x, y, \rho, \xi) \qquad \forall c > 0.$$

Proof. Define

$$I(t, x, y, \rho, \xi; \pi) = \mathbb{E}\left[\frac{\left(W_T^{m,\pi}\right)^{1-\alpha}}{1-\alpha} \left| W_t^m = x, W_t^i = y, p_t = p, \xi_{2,t} = \xi \right. \right]$$

where $\pi = \left(\pi_1^F, \pi_2^F, \pi_1^P, \pi_2^P\right)$. Assume that π^* is an optimal strategy, i.e., $I(t, x, y, \rho, \xi; \pi^*) \geq I(t, x, y, \rho, \xi; \pi)$ for all admissible strategies $\pi \in \Pi$.

Using the homogeneity of the wealth dynamics for W^m and W^i, we have that for all consumption choices $c > 0$,

$$\begin{aligned} I(t, cx, cy, \rho, \xi; \pi) &= \mathbb{E}\left[\frac{\left(W_T^{m,\pi}\right)^{1-\alpha}}{1-\alpha} \left| W_t^m = cx, W_t^i = cy, p_t = p, \xi_{2,t} = \xi \right. \right] \\ &= c^{1-\alpha} \mathbb{E}\left[\frac{\left(W_T^{m,\pi}\right)^{1-\alpha}}{1-\alpha} \left| W_t^m = x, W_t^i = y, p_t = p, \xi_{2,t} = \xi \right. \right]. \\ &= c^{1-\alpha} I(t, x, y, \rho, \xi; \pi) \end{aligned}$$

Hence,

$$\begin{aligned} I(t, cx, cy, \rho, \xi; \pi^*) &= c^{1-\alpha} I(t, x, y, \rho, \xi; \pi^*) \\ &\geq c^{1-\alpha} I(t, x, y, \rho, \xi; \pi) \\ &= I(t, cx, cy, \rho, \xi; \pi). \end{aligned}$$

for all admissible strategies $\pi \in \Pi$. This concludes the proof. $\square$

Appendix B. Description of the Numerical Method

The numerical method for calculating the optimal solution to the fund allocation problem is significantly simplified from the assumption that investor preferences are logarithmic and the game between the manager and the investor is Stackelberg with the manager as the leader. Logarithmic preferences for the investor result in myopic choices, given the fund allocations, allowing for the solution of the intertemporal problem facing the manager using dynamic programming: given a particular fund allocation, the manager knows the amount invested in the fund, and is able to calculate her expected utility. The solution is obtained backwards, starting from the terminal period when utility from terminal wealth can be directly calculated, and determining fund allocations for previous periods by maximizing expected utility for the manager.

The problem is complicated due to the number of state and choice variables. The state variables necessary to solve the problem are: the ratio of the wealth of the manager to the wealth of the investor; the level of the mean-reverting process corresponding to a component of the manager's ability; and, the percentage of the manager's wealth committed to the fund. The choice variables are the allocations in the risky assets both inside the fund and in the manager's personal account. In the most general case there are 3 state variables and 4 choice variables for each time period.

To be able to solve the problem we discretize time as well as the state variables and approximate the evolution of the asset prices by a binomial model, described below. For each time period we estimate the value function on a regular grid of the state variables: the level of the mean-reverting process; the percentage of the manager's wealth committed to the fund; and the ratio of the wealth of the investor to the wealth of the manager. For each point in the state space grid, we need to estimate the discounted expected value of the value function for the following period at points in the state space that are determined by values of the choice variables and the evolution of the asset prices. This calculation requires the ability to approximate the value function at points in the state variable space other than the grid points. To estimate the value function for values of the state variables that do not match the values of the grid points we follow a technique described in sections 3.2 and 3.6.1 of “Numerical Recipes”, 3rd edition, by Press, Teukolsky, Vetterling, and Flannery (2007): for each point we want to approximate the value function at we identify a block of nearby grid points. The number of points used depends on the order of the approximation we want to achieve. We use linear interpolation in the directions of the level of the mean-reverting process and the percentage of the manager’s wealth committed to the fund, and cubic interpolation in the ratio of the wealth of the investor to the wealth of the manager. This implies that we use $2 \times 2 \times 4$ grid points for each approximation. The approximate value is calculated by successively applying Neville’s algorithm along each dimension in the state space.[27]

We test convergence of our algorithm by trying different sizes of discretization steps for the state variable grid and verifying that the results are not significantly influenced by the choice of discretization step.

Binomial Approximation

We implement the Jarrow-Rudd (JR) version of the binomial model in the sense that for each random factor, there are only two future states, up and down, with the same probability $1/2$. The up and down magnitudes are chosen in order to match the first and the second moment of the process at time $t + dt$. With the two factors, there are four possible states, $\{uu, ud, du, dd\}$, each with probability $1/4$.

[27] For each dimension, given a set of m grid points, Neville’s algorithm computes the value of an interpolating polynomial of degree $m-1$ at any point by taking successive differences, but without explicitly estimating the polynomial coefficients — see section 3.2 in “Numerical Recipes”, 3rd edition, by Press, Teukolsky, Vetterling, and Flannery (2007).

The approximations are given by

$$S_{1,t+dt} = \begin{cases} S_{1,t} \exp\left\{ (r + \sigma_{11}\xi_1 - \frac{1}{2}(\sigma_{11})^2)dt + \sigma_{11}\sqrt{dt} \right\}, \text{ in states } uu, ud \\ S_{1,t} \exp\left\{ (r + \sigma_{11}\xi_1 - \frac{1}{2}(\sigma_{11})^2)dt - \sigma_{11}\sqrt{dt} \right\}, \text{ in states } du, dd \end{cases}$$

$$S_{2,t+dt} = \begin{cases} S_{2,t} \exp\left\{ (r + \sigma_{21}\xi_1 + \sigma_{11}\xi_{2,t} - \frac{1}{2}(\sigma_{21})^2 + (\sigma_{22})^2)dt + \sigma_{21}\sqrt{dt} + \sigma_{22}\sqrt{dt} \right\}, \text{in state } uu \\ S_{2,t} \exp\left\{ (r + \sigma_{21}\xi_1 + \sigma_{11}\xi_{2,t} - \frac{1}{2}(\sigma_{21})^2 + (\sigma_{22})^2)dt + \sigma_{21}\sqrt{dt} - \sigma_{22}\sqrt{dt} \right\}, \text{in state} ud \\ S_{2,t} \exp\left\{ (r + \sigma_{21}\xi_1 + \sigma_{11}\xi_{2,t} - \frac{1}{2}(\sigma_{21})^2 + (\sigma_{22})^2)dt - \sigma_{21}\sqrt{dt} + \sigma_{22}\sqrt{dt} \right\}, \text{in state } du \\ S_{2,t} \exp\left\{ (r + \sigma_{21}\xi_1 + \sigma_{11}\xi_{2,t} - \frac{1}{2}(\sigma_{21})^2 + (\sigma_{22})^2)dt - \sigma_{21}\sqrt{dt} - \sigma_{22}\sqrt{dt} \right\}, \text{in state } dd \end{cases}$$

and

$$\xi_{2,t+dt} = \begin{cases} \xi_{2,t}e^{-\lambda dt} + \bar{\xi}_2(1 - e^{-\lambda dt}) + \psi\sqrt{\frac{\bar{\xi}_2(1 - e^{-\lambda dt}) + 2\xi_{2,t}(e^{-\lambda dt} - e^{-2\lambda dt})^2}{\lambda\bar{\xi}_2}} & \text{in states } uu, du \\ \xi_{2,t}e^{-\lambda dt} + \bar{\xi}_2(1 - e^{-\lambda dt}) - \psi\sqrt{\frac{\bar{\xi}_2(1 - e^{-\lambda dt}) + 2\xi_{2,t}(e^{-\lambda dt} - e^{-2\lambda dt})^2}{\lambda\bar{\xi}_2}} & \text{in states } ud, dd \end{cases}$$

Appendix C. Learning Managerial Ability

We assume that both the fund and the investor have access to a risk-free bond with price S_0, and, in addition, the fund has access to a risky investment opportunity with price S_1. The dynamics of the prices of the two investments are given by

$$\frac{dS_t^0}{S_t^0} = rdt$$
$$\frac{dS_t^1}{S_t^1} = (r + \sigma\xi)dt + \sigma dB_t$$

where r is the risk-free interest rate, σ the volatility of the risky investment, and ξ the true Sharpe ratio of the risky investment. We assume that ξ is unknown to both the manager and the investor, and that they both share a common prior on the value of ξ, which is normally distributed, with mean a_0 and variance v_0. The manager chooses the proportion of the fund, π, to be invested in the risky asset. The dynamics of the fund value F is given by

$$\frac{dF_t}{F_t} = (1-\pi_t)\frac{dS_t^0}{S_t^0} + \pi_t \frac{dS_t^1}{S_t^1} - \gamma dt = (r - \pi_t \sigma\theta - \gamma)dt + \sigma\pi_t dB_t \tag{C1}$$

where γ is the proportional fee for investing in the fund. We assume that the allocation in the risky investment opportunity, π, is observable by the investor, and separate the observable terms in Equation (C1)

$$d\Omega_t = (\frac{dF_t}{F_t} - (r-\gamma)dt)/\pi_t = \xi\sigma dt + \sigma dB_t$$

Learning implies that, once the fund return is observed, the investor and manager update their beliefs about the Sharpe Ratio ξ from the observed changes in the variable Ω_t. From filtering theory we have that the posterior distribution of ξ is normal at every point of time, and that the posterior mean and variance of ξ, denoted by a_t and v_t respectively, evolve according to the following differential equation system:[28]

$$\begin{aligned} da_t &= \frac{v_t}{\sigma}[d\Omega_t - a_t \sigma dt] \\ dv_t &= -v_t^2 dt \end{aligned}$$

where the posterior variance, v_t, can be solved explicitly

$$v_t = \frac{v_0}{v_0 t + 1}. \tag{C2}$$

[28]See Dangl, Wu, and Zechner (2008) and Lipster and Shiryayev (1978) for further discussion and proof.

The remaining formulation, describing the dynamics of the investor's wealth, the manager's wealth, and the manager's choice of the fund and personal portfolio allocations remain the same.

To numerically solve the problem with learning managerial ability, we need to add a state variable to the formulation of the dynamic program for the problem without learning: the invetor's and manager's prior of the manager's ability. This additional state variable makes the problem of a manager that has access to a risky asset with a constant Sharpe ratio but whose true ability is learned by observing fund performance, as difficult as the problem faced by a manager whose ability is known, but mean-reverting. Due to the difficulty of solving the general problem with an additional state variable, we have only implemented the case of learning managerial ability for a manager whose ability is constant.

Appendix D. Competing funds

Market Dynamics

The investor has access to a money market account with constant return r, as well as a risky asset that corresponds to the market portfolio. The investor can also allocate his wealth to the managed funds. The dynamics of the returns of the assets available to the investor are described by

$$
\begin{aligned}
\frac{dS_{0,t}}{S_{0,t}} &= rdt \\
\frac{dS_{m,t}}{S_{m,t}} &= (r + \sigma_m \xi_m)dt + \sigma_m dB_{m,t}
\end{aligned}
$$

where S_0 corresponds to the amount in the money market account, S_m the value of a share of the market portfolio, and ξ_m the Sharpe ratio for the market portfolio.

Managerial Ability and Value of Assets under Management

In addition to the money market account, also available to the retail investor, we allow each fund access to investment opportunities associated with their managers' investment abilities. We limit each fund to access to a single risky asset, corresponding to either constant or mean-reverting managerial ability.

The returns of the strategies corresponding to managerial ability available to each fund are described by

$$\frac{dS_{1,t}^k}{S_{1,t}^k} = rdt + \sigma^k \left(\xi_t^k dt + dB_t^k \right), k = 1,2$$

where S_1^1 denotes the value of the investment opportunity available to the first fund, and S_1^2 denotes the value of the investment opportunity available to the second fund. The parameters $\sigma^k, k = 1,2$, are assumed to be constant, while the Brownian motions B^1, B^2 are assumed to be correlated with correlation ρ. The Sharpe ratio of the investment opportunity of each fund may be constant, corresponding to a manager with constant ability, or mean-reverting with CIR dynamics.

The value of assets under management of each fund F^1, F^2 follow dynamics similar to those in Equation (2).

$$\frac{dF_{k,t}}{F_{k,t}} = (1 - \pi_{k,t}) \frac{dS_{m,t}}{S_{m,t}} + \pi_{k,t} \frac{dS_{F,t}^k}{S_{F,t}^k} - \gamma_k dt, \quad k = 1,2$$

where $\pi^k, k = 1,2$ are the fund allocation percentages in the risky assets, and $\gamma^k, k = 1,2$ are the percentage fees paid by the investor.

Investor Wealth

The dynamics of the wealth of the investor, W^i is given by

$$\frac{dW_t^i}{W_t^i} = \left(1 - \phi_{m,t} - \sum_{k=1}^{2} \phi_{k,t} \right) \frac{dS_{0,t}}{S_{0,t}} + \phi_{m,t} \frac{dS_{m,t}}{S_{m,t}} + \sum_{k=1}^{2} \phi_{k,t} \frac{dF_{k,t}}{F_{k,t}}, \phi_{k,t} \geq 0, \quad k = 1,2$$

where ϕ^1, ϕ^2 are the percentages of the investor's wealth invested in the each of the funds, ϕ_m the percentage of the investor's wealth invested in the market portfolio, and F^1, F^2 the values of each of the funds.

Optimization Problem

Similar to the single fund case, we assume that the interaction between the fund managers and the investor follows a Stackelberg differential game, with the fund managers being the leaders. The investor maximizes expected utility from terminal wealth and has logarithmic preferences. Given the fund compositions, the investor's optimal allocation strategy to each fund is given by

$$\phi = (\Sigma\Sigma^T)^{-1}(\mu - rI)$$

where

$$\mu - rI = \begin{pmatrix} \sigma_m \xi_m \\ \pi_1 \sigma_1 \xi_1 + (1-\pi_1)\sigma_m \xi_m - \gamma_1 \\ \pi_2 \sigma_2 \xi_2 + (1-\pi_2)\sigma_m \xi_m - \gamma_2 \end{pmatrix}$$

and $\Sigma\Sigma^T$ the covariance matrix of the returns for the market portfolio and the returns for the idiosyncatic strategies available to each fund.

The fund managers are assumed to act simultaneously in their choice of asset allocations. They chose fund allocations such that neither manager can improve their expected utility by making small changes; i.e., reaching a Nash equilibrium.[29]

We assume that the managers choose allocations that maximize investor participation.[30]

[29]Given the complexity of the problem, multiple equilibria might exist. We concentrate in symmetric situations and focus on the symmetric equilibrium.

[30]For example, similar to the case of a single manager, the manager choose allocations that maximize investor participation when their personal wealth is relatively small compared to the wealth of the investor.

The first order conditions that maximize investor participation are

$$\frac{\partial \phi_1}{\partial \pi_1} = 0$$
$$\frac{\partial \phi_2}{\partial \pi_2} = 0$$

The solution of the first order conditions is given in Lemma 4.1. The corollaries follow from the Lemma.

Appendix E. Numerical Procedure to Find the Nash Equilibrium

In what follows we briefly explain the numerical procedure to find the Nash equilibrium to the problem with two competing mutual funds. The basic idea is iteration: for each time time, we improve the allocation strategies of each manager separately until no further improvements were possible.

We choose a tolerance $\varepsilon > 0$ and the initial allocation $\pi^1(0)$, and find the solution to the optimization problem, $\forall j \geq 1$,

$$\begin{aligned} \pi^2(j) \in \arg\max_{\pi^2} \mathbb{E}[V^2(t+dt, W^{1,m}_{t+dt}, W^{2,m}_{t+dt}, W^i_{t+dt}, \xi^1_{t+dt}, \xi^2_{t+dt}) | W^{1,m}_t = x, \\ W^{2,m}_t = y, W^i_t = z, \xi^1_t = \xi^1, \xi^2_t = \xi^2, \pi^1(j-1)] \end{aligned}$$

Then we use $\pi^2(j-1)$ as the feedbacks in the other optimization problem

$$\begin{aligned} \pi^1(j) \in \arg\max_{\pi^1} \mathbb{E}[V^1(t+dt, W^{1,m}_{t+dt}, W^{2,m}_{t+dt}, W^i_{t+dt}, \xi^1_{t+dt}, \xi^2_{t+dt}) | W^{1,m}_t = x, \\ W^{2,m}_t = y, W^i_t = z, \xi^1_t = \xi^1, \xi^2_t = \xi^2, \pi^1(j)] \end{aligned}$$

For $j > 1$, the iterative procedure stops when

$$\sum_{k=1,2} (\pi^k(j) - \pi^k(j-1))^2 < \varepsilon$$

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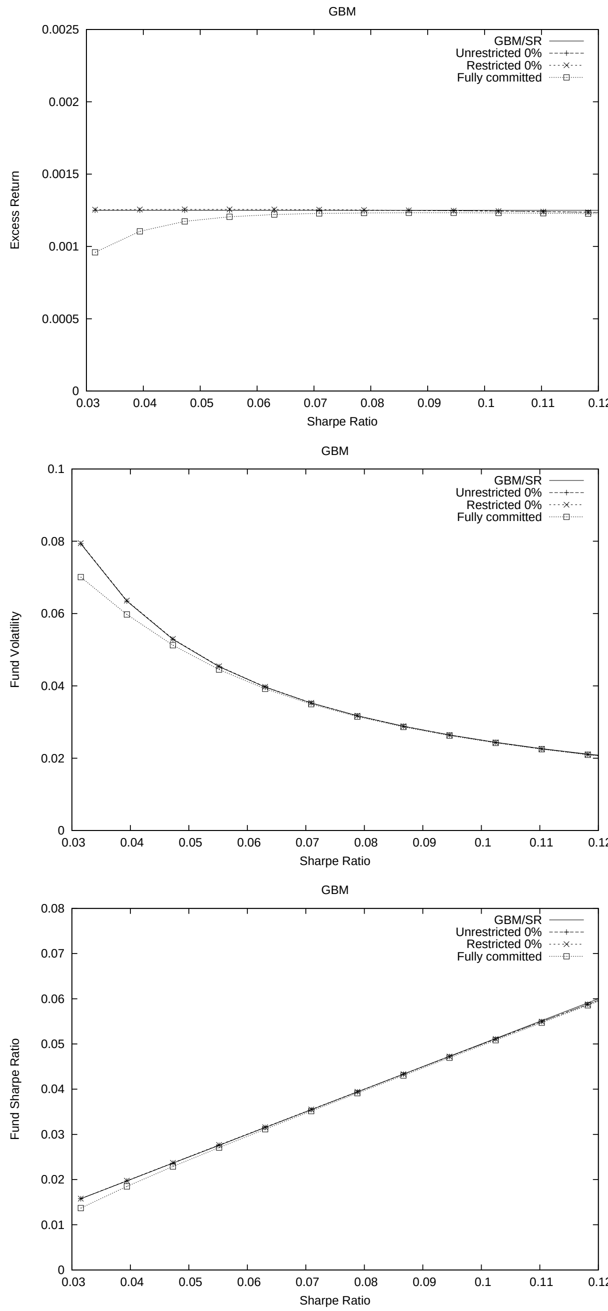


Figure 1. This figure presents the fund expected excess returns (per month, top panel), idiosyncratic volatility (monthly, middle panel), and Sharpe ratio (monthly, bottom panel), for managers of different levels of ability, represented by the Sharpe ratio. The fund's opportunity set consists of a risky asset that follows geometric Brownian motion with volatility $\sigma_{11} = 0.0436$ monthly, and a risk-free asset with interest rate $r = 0.4167\%$ per month. The ratio of the manager's wealth to the investor's wealth is 1%. The lines "GBM/SR" correspond to the expected excess return, idiosyncratic volatility, and Sharpe ratio at the fund allocation chosen by a manager that maximizes investor participation. The line "Unrestricted 0%" corresponds to a manager that does not commit any of her wealth to the fund and who is unrestricted from accessing the risky asset in her personal account. The line "Restricted 0%" corresponds to a manager that does not commit any of her wealth to the fund but who is restricted from accessing the risky asset in her personal account. The line "Fully committed" corresponds to a manager that commits all her wealth to the fund.

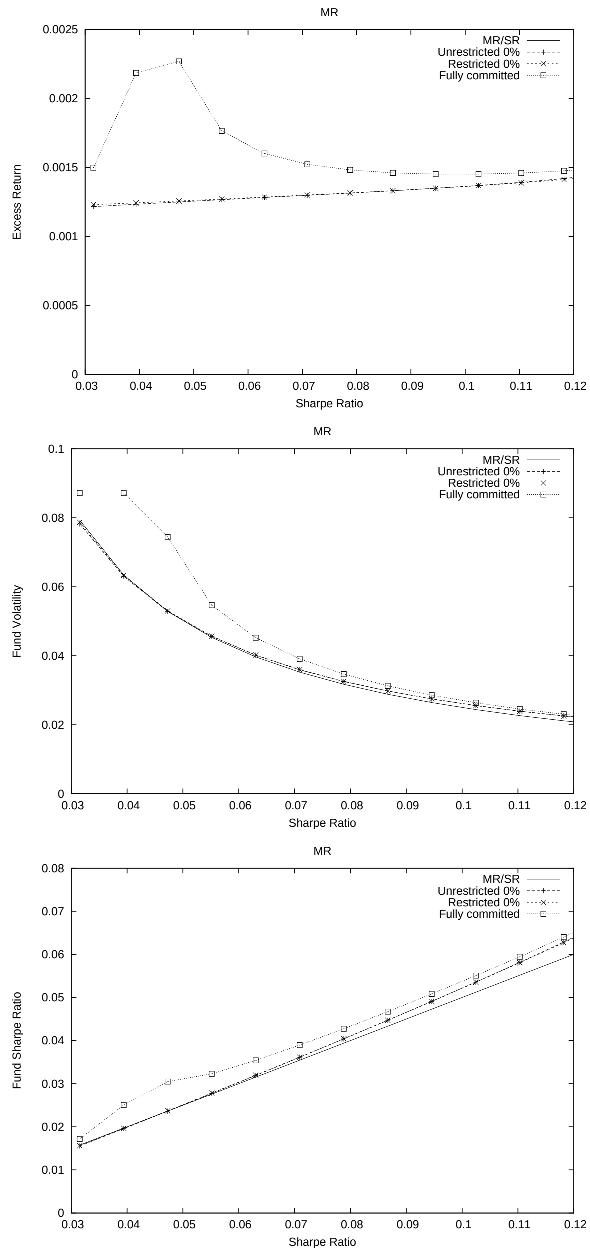


Figure 2. This figure presents the fund expected excess returns (per month, top panel), idiosyncratic volatility (monthly, middle panel), and Sharpe ratio (monthly, bottom panel), for managers of different initial levels of ability, represented by the Sharpe ratio. The fund's opportunity set consists of a risky asset with mean-reverting Sharpe ratio with volatility $\sigma_{21} = 0, \sigma_{22} = 0.0436$, and a risk-free asset with interest rate $r = 0.4167\%$ per month. The mean-reversion rate is $\lambda = 0.016$, and the volatility of the Sharpe ratio is $\psi = -0.0189$. The ratio of the manager's wealth to the investor's wealth is 1%. The lines "MR/SR" correspond to the expected excess return, idiosyncratic volatility, and Sharpe ratio at the fund allocation chosen by a manager that maximizes investor participation. The line "Unrestricted 0%" corresponds to a manager that does not commit any of her wealth to the fund and who is unrestricted from accessing the risky asset in her personal account. The line "Restricted 0%" corresponds to a manager that does not commit any of her wealth to the fund but who is restricted from accessing the risky asset in her personal account. The line "Fully committed" corresponds to a manager that commits all her wealth to the fund.

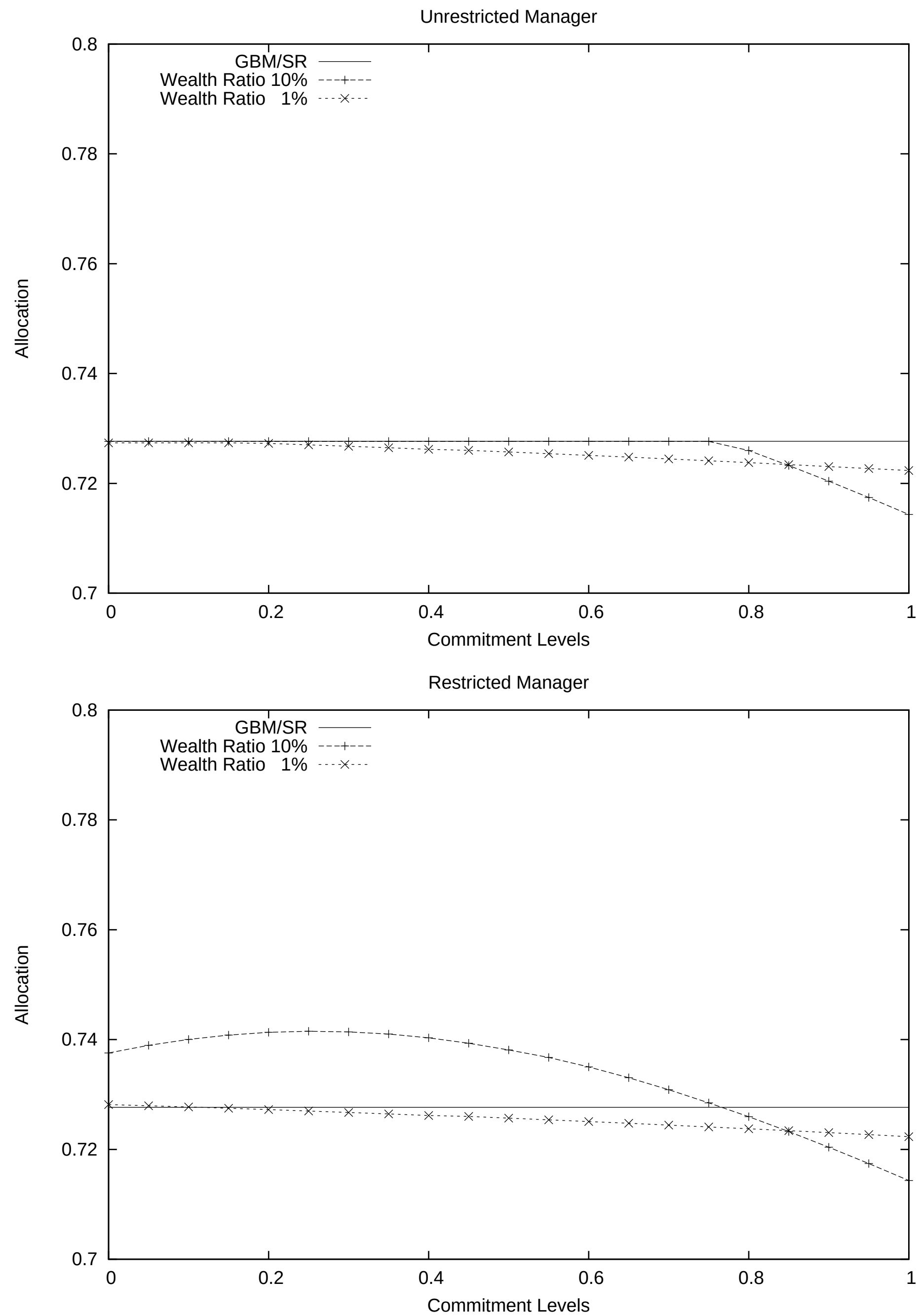


Figure 3. This figure presents the fund asset allocations for different levels of commitment of managerial wealth to the fund. The fund's opportunity set consists of a risky asset that follows geometric Brownian motion with volatility $\sigma_{11} = 0.0436$ monthly and Sharpe ratio $\xi_1 = 0.0788$ per month, and a risk-free asset with interest rate $r = 0.4167\%$ per month. In addition to the fund allocations, the lines "GBM/SR" correspond to the allocations that would maximize investment in the fund. In the top panel we consider the case of a manager that is unrestricted from accessing the risky asset in the personal account, while in the bottom panel the manager is restricted from accessing the risky asset in her personal account. The lines "Wealth Ratio 1%" and "Wealth Ratio 10%" correspond to managers whose initial wealth is 1% and 10% of the investor's wealth, respectively.

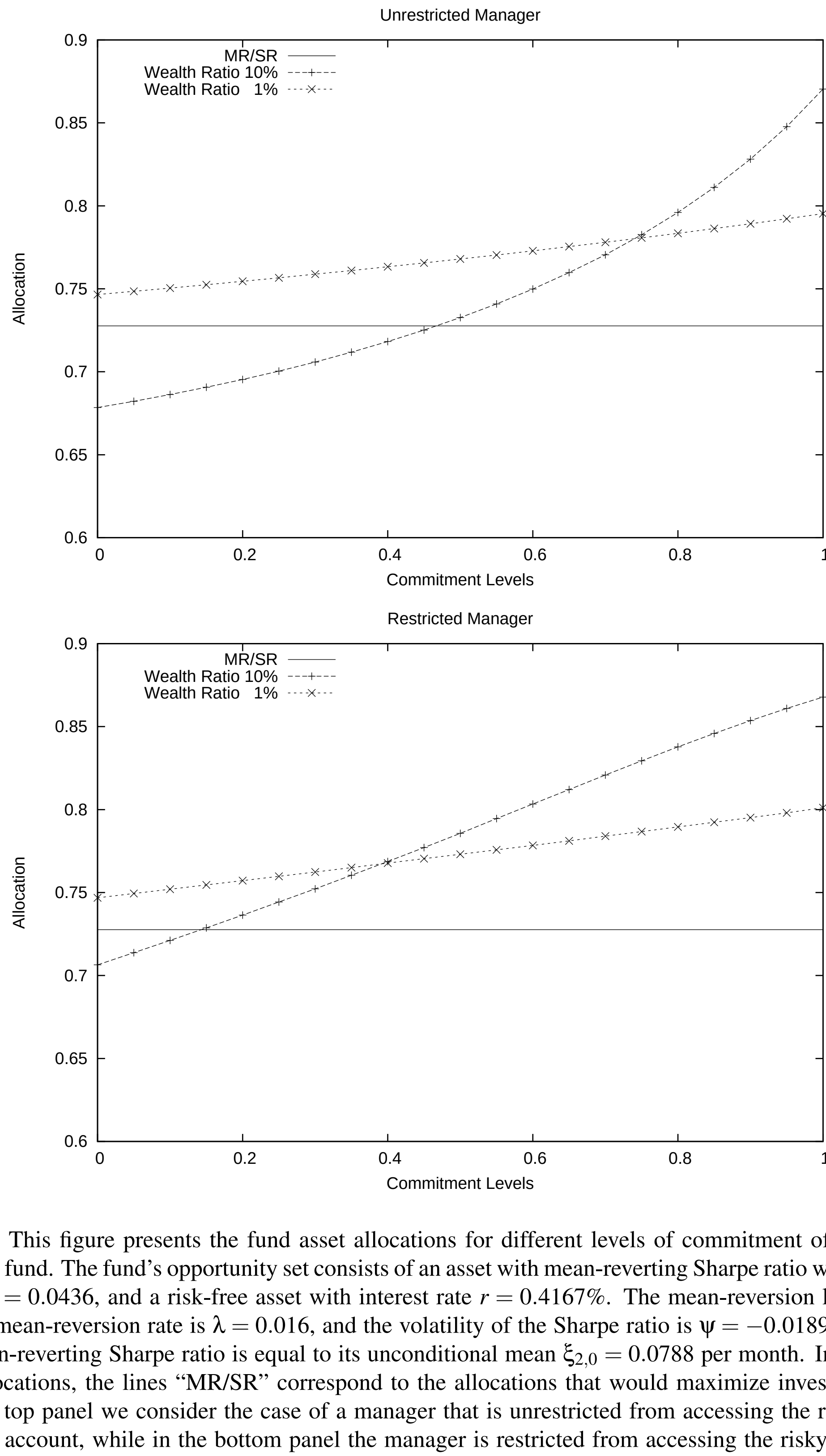


Figure 4. This figure presents the fund asset allocations for different levels of commitment of managerial wealth to the fund. The fund's opportunity set consists of an asset with mean-reverting Sharpe ratio with volatility $\sigma_{21} = 0, \sigma_{22} = 0.0436$, and a risk-free asset with interest rate $r = 0.4167\%$. The mean-reversion level is $\bar{\xi}_2 = 0.0788$, the mean-reversion rate is $\lambda = 0.016$, and the volatility of the Sharpe ratio is $\psi = -0.0189$. The initial level of mean-reverting Sharpe ratio is equal to its unconditional mean $\xi_{2,0} = 0.0788$ per month. In addition to the fund allocations, the lines "MR/SR" correspond to the allocations that would maximize investment in the fund. In the top panel we consider the case of a manager that is unrestricted from accessing the risky asset in her personal account, while in the bottom panel the manager is restricted from accessing the risky asset in her personal account. The lines "Wealth Ratio 1%" and "Wealth Ratio 10%" correspond to managers whose initial wealth is 1% and 10% of the investor's wealth, respectively.

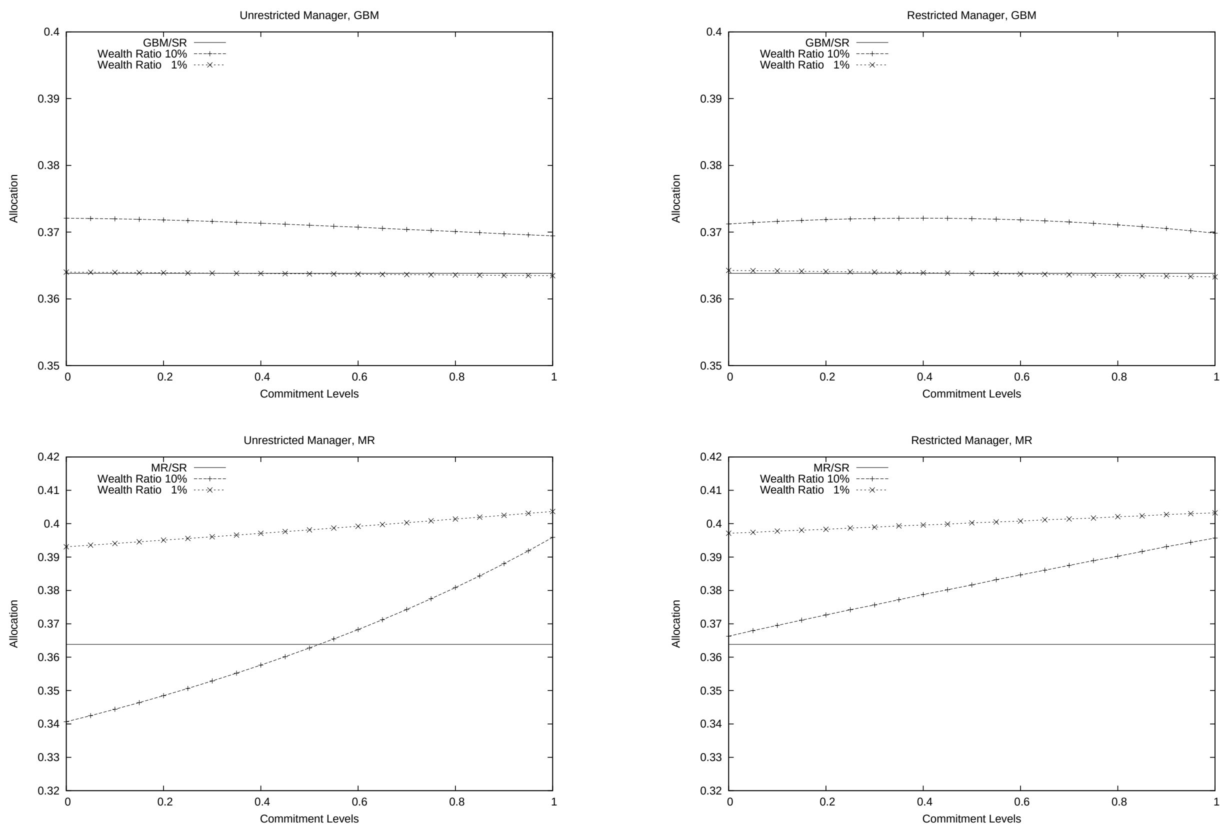


Figure 5. This figure presents the asset allocations for different levels of commitment of managerial wealth to the fund. The fund's opportunity set consists of two risky assets and the riskless asset. The first risky asset follows geometric Brownian motion with expected return $\xi_1 = 0.0788$ per month, and volatility $\sigma_{11} = 0.0436$ monthly. The second risky asset has a mean-reverting Sharpe ratio with volatility $\sigma_{21} = 0, \sigma_{22} = 0.0436$. The mean-reversion level is $\bar{\xi}_2 = 0.0788$ per month, the mean-reversion rate is $\lambda = 0.016$ per month, and the volatility of the Sharpe ratio is $\psi = -0.0189$ monthly. The riskless interest rate is $r = 0.4167\%$ per month. The initial level of mean-reverting Sharpe ratio is equal to its unconditional mean $\xi_{2,0} = 0.0788$ per month. In addition to the fund allocations, the lines "MR/SR" and "GBM/SR" correspond to the allocations that would maximize investment in the fund. The lines "Wealth Ratio 1%" and "Wealth Ratio 10%" correspond to managers whose initial wealth is 1% and 10% of the investor's wealth, respectively. In the top row we consider the case of a manager that is unrestricted from accessing the risky asset in her personal account, while in the bottom row the manager is restricted from accessing the risky asset in her personal account. The figures on the left present the allocation in the asset the follows geometric Brownian motion, while the figures on the right present the allocation in the asset with the mean-reverting Sharpe ratio.

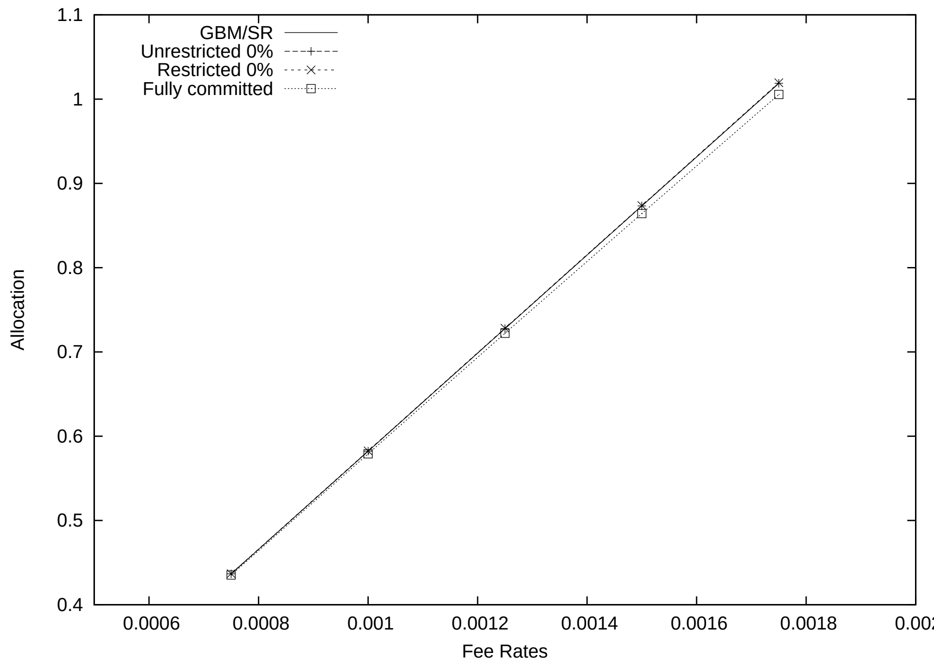


Figure 6. This figure presents the fund asset allocations for different levels of fees collected by the manager. The fund's opportunity set consists of a risky asset that follows geometric Brownian motion with volatility $\sigma_{11} = 0.0436$ monthly, and a risk-free asset with interest rate $r = 0.4167\%$ per month. The initial ratio of the manager's wealth to the investor's wealth is 1%. In addition to the fund allocations, the line "GBM/SR" corresponds to the allocation that would maximize investment in the fund. The line "Unrestricted 0%" corresponds to a manager that does not commit any of her wealth to the fund and who is unrestricted from accessing the risky asset in her personal account. The line "Restricted 0%" corresponds to a manager that does not commit any of her wealth to the fund but who is restricted from accessing the risky asset in her personal account. The line "Fully committed" corresponds to a manager that commits all her wealth to the fund.

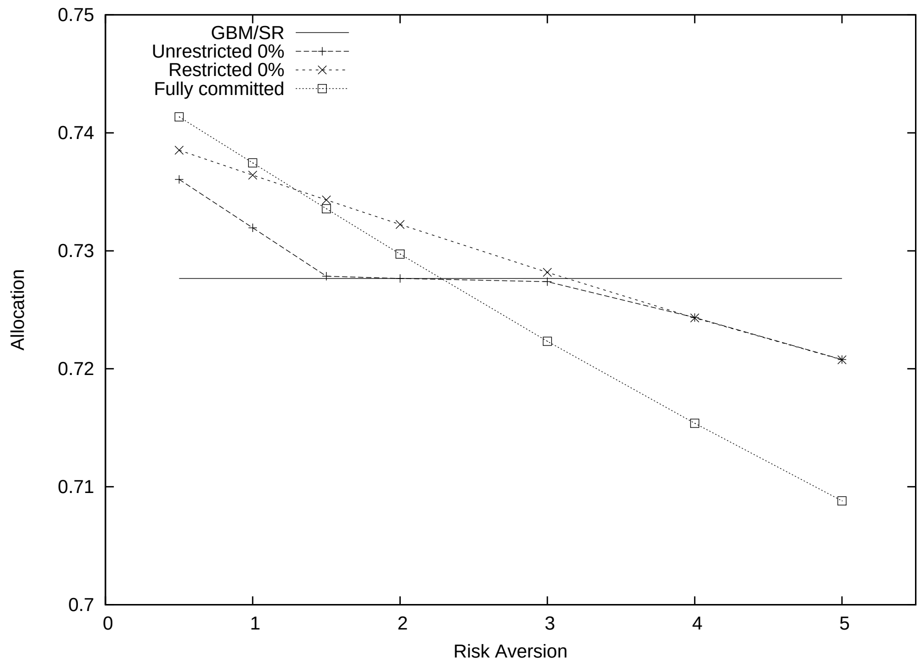


Figure 7. This figure presents the fund asset allocations for managers with different levels of risk aversion. The fund's opportunity set consists of a risky asset that follows geometric Brownian motion with volatility $\sigma_{11} = 0.0436$ monthly, and a risk-free asset with interest rate $r = 0.4167\%$ per month. The initial ratio of the manager's wealth to the investor's wealth is 1%. In addition to the fund allocations, the line "GBM/SR" corresponds to the allocation that would maximize investment in the fund. The line "Unrestricted 0%" corresponds to a manager that does not commit any of her wealth to the fund and who is unrestricted from accessing the risky asset in her personal account. The line "Restricted 0%" corresponds to a manager that does not commit any of her wealth to the fund but who is restricted from accessing the risky asset in her personal account. The line "Fully committed" corresponds to a manager that commits all her wealth to the fund.

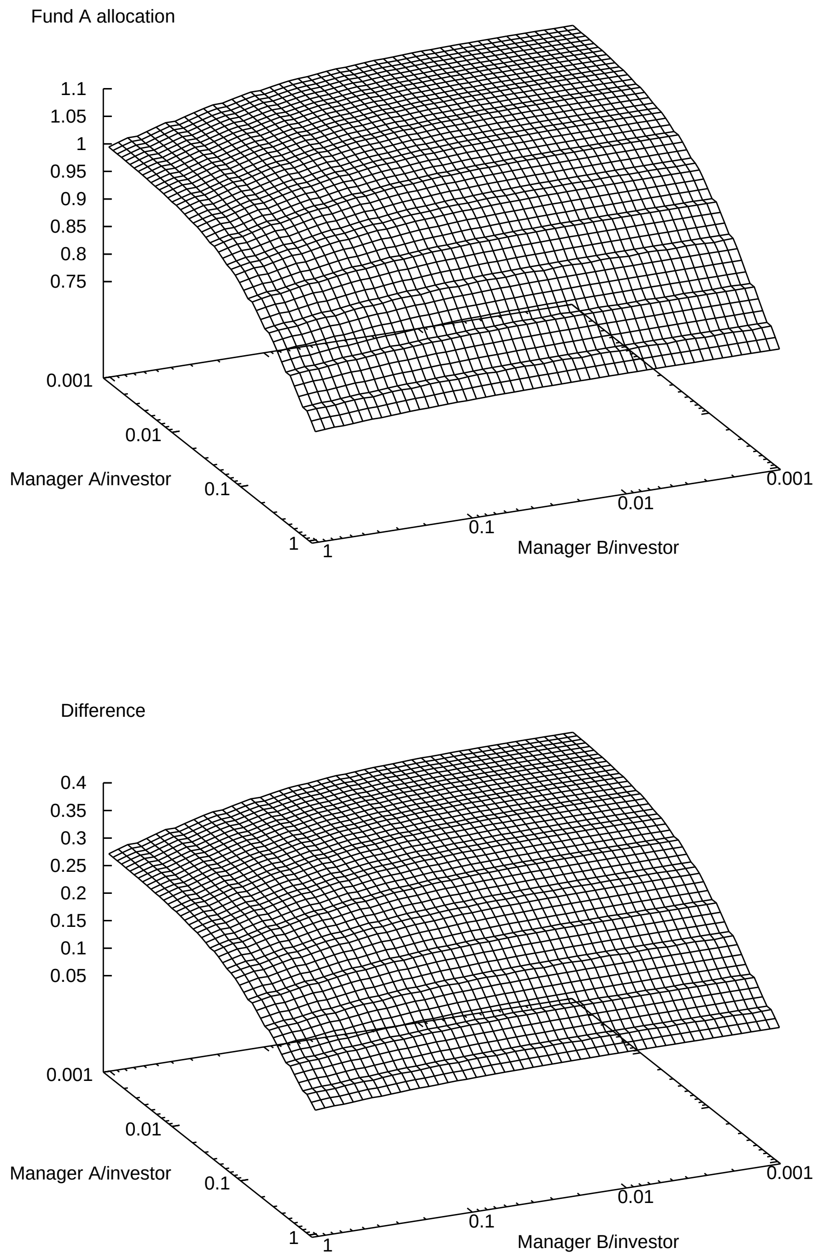
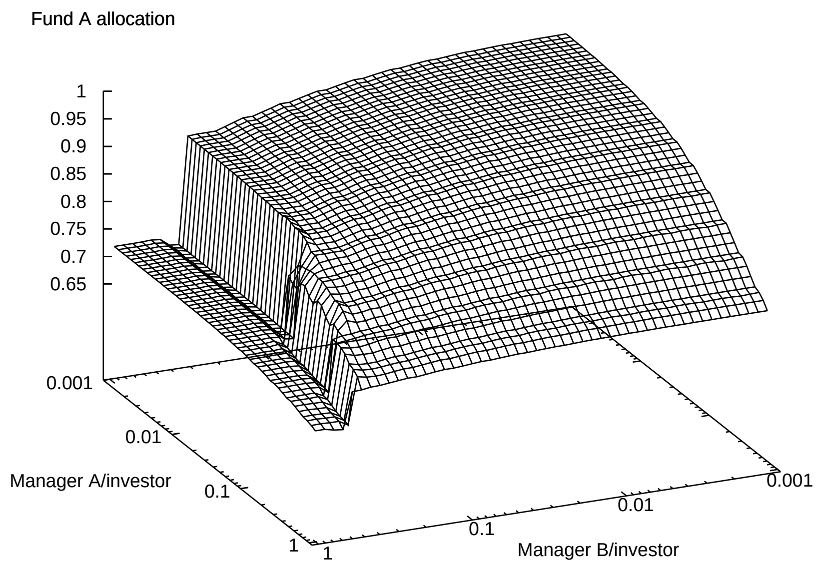


Figure 8. This figure presents the asset allocation for the case of two competing funds. Both managers invest all their wealth in the fund. Each fund has access to an investment opportunity that follows geometric Brownian motion with the same Sharpe ratios $\xi_t^1 = \xi_t^2 = 0.0788$ per month, and volatility $\sigma^k = 0.0436, k = 1, 2$ monthly. The risk-free rate is $r = 0.4167\%$. The correlation between the two assets is $\rho = 50\%$. The top panel displays the allocation of Fund A in the risky investment opportunity as a function of the ratios of the wealth of each manager with respect to the wealth of the investor, while the bottom panel displays the difference in the allocation of each fund to the risky investment opportunity between the cases of competing and monopolistic managers.



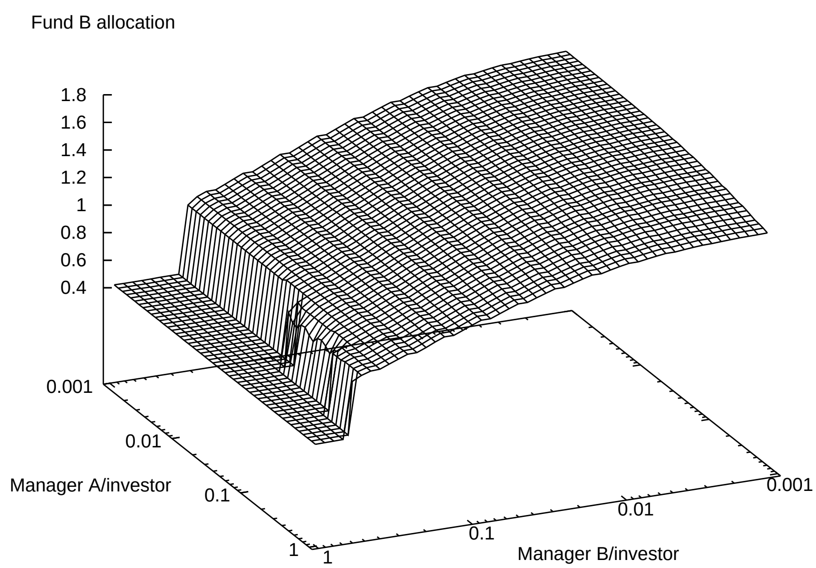


Figure 9. This figure presents the asset allocation for the case of two competing funds. Both managers invest all their wealth in the fund. Each fund has access to an investment opportunity that follows geometric Brownian motion. The Sharpe ratios of Fund A is higher than the Sharpe ratio of Fund B, with values $\xi_t^1 = 0.0788, \xi_t^2 = 0.0597$ per month, and volatility $\sigma^k = 0.0436, k = 1,2$ monthly. The risk-free rate is $r = 0.4167\%$. The correlation between the two assets is $\rho = 50\%$. The top panel displays the allocation of Fund A in the risky investment opportunity as a function of the ratios of the wealth of each manager with respect to the wealth of the investor, while the bottom panel presents the allocation of Fund B.

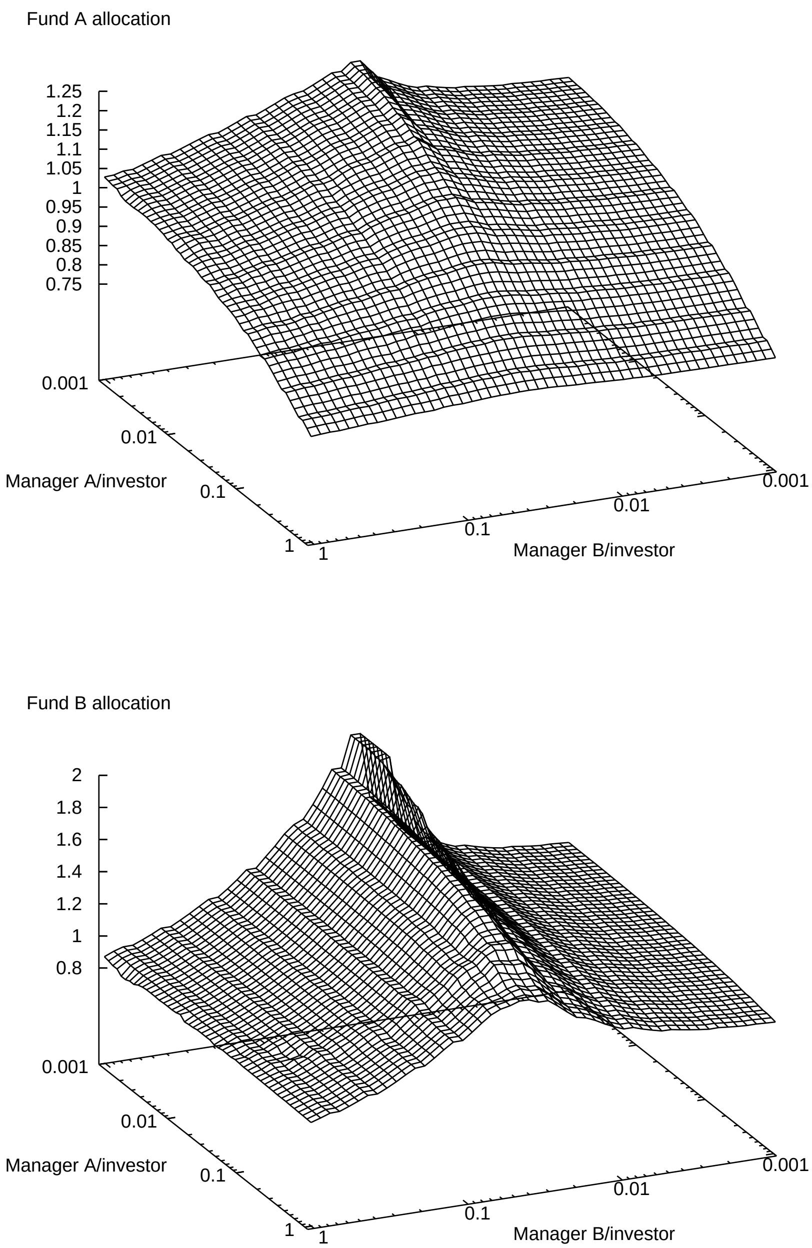


Figure 10. This figure presents the asset allocation for the case of two competing funds. Both managers invest all their wealth in the fund. Fund A has access to an investment opportunity that follows geometric Brownian motion with Sharpe ratio $\xi_t^1 = 0.0788$ per month, and volatility $\sigma^1 = 0.0436$ monthly. Fund B has access to an investment opportunity with mean-reverting Sharpe ratio, with volatility $\sigma^2 = 0.0436$ monthly, the level of the mean-reverting Sharpe ratio is $\bar{\xi}^2 = 0.0788$ per month, mean-reversion rate $\lambda^2 = 0.016$ per month, $\psi^2 = -0.0189$ monthly. The level of the Sharpe ratio is at its mean level $\xi_0^2 = 0.0788$ per month. The risk-free rate is $r = 0.4167\%$. The correlation between the two assets is $\rho = 50\%$. The top panel displays the allocation of Fund A in the risky investment opportunity as a function of the ratios of the wealth of each manager with respect to the wealth of the investor, while the bottom panel presents the allocation of Fund B.

Table 1
Parameter Values - Basecase

This table presents the basecase parameter values: r is the interest rate, T is the time horizon, ξ_1 is the Sharpe ratio associated with the risky asset that represents the constant part of a manager's ability, σ_{11} is the volatility of the risky asset that represents the constant part of a manager's ability, γ is the proportional fee charged to the investor, m^+ is the margin requirement for long positions, m^- is the margin requirement for short positions, and α is the level of risk aversion of the manager.

Parameter	Value
r	0.4167% per month
T	12 months
ξ_1	0.0788 per month
σ_{11}	0.0436 (monthly)
γ	0.125% per month
m^+	50%
m^-	150%
α	3

Table 2
Wealth Equivalents

This table presents wealth equivalents for managers and investors, comparing each case to the case of a manager that is unrestricted from investing in the risky asset in her personal account. The manager has access to an investment opportunity that follows geometric Brownian motion (GBM), with the Sharpe ratio given in the table, and volatility $\sigma_{11} = 0.0436$ monthly. The remaining parameters are given in Table 1. The column labeled F/U-Manager measures the wealth equivalent difference between a manager who commits her entire wealth to the fund she manages with a manager that does not commit any of her wealth to the fund and is unrestricted from accessing the risky asset in her personal account. The column labeled R/U-Manager measures the wealth equivalent difference between a manager who does not commit any of her entire wealth to the fund she manages but is restricted from accessing the risky asset in her personal account with a manager that does not commit any of her wealth to the fund and is unrestricted from accessing the risky asset in her personal account. The column labeled F/U-Investor measures the wealth equivalent difference between an investor that can invest in a fund managed by a manager who commits her entire wealth to the fund she manages with an investor that can invest in a fund managed by a manager that does not commit any of her wealth to the fund and is unrestricted from accessing the risky asset in her personal account. The column labeled R/U-Investor measures the wealth equivalent difference between an investor that can invest in a fund managed by a manager who does not commit any of her entire wealth to the fund she manages but is restricted from accessing the risky asset in her personal account an investor that can invest in a fund managed by a manager that does not commit any of her wealth to the fund and is unrestricted from accessing the risky asset in her personal account. The Sharpe ratio for the risky asset is in monthly units and is measured in percentage points.

Sharpe ratio	F/U - Manager	R/U - Manager	F/U - Investor	R/U - Investor
3.2	-3.9%	-0.1%	0.0%	0.0%
3.9	-1.8%	-0.2%	0.0%	0.0%
4.7	-0.7%	-0.3%	0.0%	0.0%
5.5	-0.3%	-0.3%	0.0%	0.0%
6.3	-0.1%	-0.4%	0.0%	0.0%
7.1	-0.1%	-0.4%	0.0%	0.0%
7.9	-0.1%	-0.5%	0.0%	0.0%
8.7	-0.2%	-0.6%	0.0%	0.0%
9.5	-0.3%	-0.6%	0.0%	0.0%
10.2	-0.4%	-0.7%	0.0%	0.0%
11.0	-0.5%	-0.8%	0.0%	0.0%
11.8	-0.7%	-0.9%	0.0%	0.0%
12.6	-0.8%	-1.0%	0.0%	0.0%

Table 3
Parameter Values - Mean Reverting

This table presents the paratemer values for a manager with mean-reverting ability: r is the interest rate, T is the time horizon, σ_{22} corresponds to the volatility of the risky asset that represents the mean-reverting part of the manager's ability, λ is the mean-reversion rate of the Sharpe ratio associated with the mean-reverting part of the manager's ability, $\bar{\xi}_2$ is the long term level of the mean-reverting part of the manager's ability, ψ corresponds to the volatility of the Sharpe ratio of the asset that represents the mean-reverting part of the manager's ability, γ is the proportional fee charged to the investor, m^+ is the margin requirement for long positions, m^- is the margin requirement for short positions, and α is the level of risk aversion of the manager.

Parameter	Value
r	0.4167% per month
T	12 months
σ_{22}	0.0436 (monthly)
λ	0.016 per month
$\bar{\xi}_2$	0.0788 per month
ψ	-0.0189 (monthly)
γ	0.125% per month
m^+	50%
m^-	150%
α	3

Table 4
Fund Allocation vs. Correlation and Ability Level

This table presents the fund allocations as the correlation between the investment opportunities available to the manager changes. The first investment opportunity follows geometric Brownian motion (GBM), while the second investment opportunity has a Sharpe ratio with a mean-reverting component (MR). The parameters are the same as in Table 1. The "Unrestricted" manager does not commit any of her wealth to the fund and is unrestricted from accessing the risky asset in her personal account. The "Restricted" manager does not commit any of her wealth to the fund but is restricted from accessing the risky asset in her personal account. The "Fully committed" manager commits all her wealth to the fund. The initial level of the mean-reverting component is $\xi_{2,0}$, while the long-term level is $\bar{\xi}_2 = 0.0788$ per month. The columns GBM and MR present the optimal fund allocations in the GBM and MR assets, while the columns GBM/SR and MR/SR present the fund allocations that would maximize investor participation. The columns H-GBM and H-MR present the hedging demands.

Manager	$\xi_{2,0}$	correlation	GBM	MR	GBM/SR	MR/SR	H-GBM	H-MR
		-50 %	71.8	35.1	70.2	33.2	1.6	1.9
	0.051	0 %	51.2	35.1	51.2	33.2	0.0	1.9
		50 %	30.5	35.1	32.1	33.2	-1.6	1.9
		-50 %	59.3	39.3	57.2	36.4	2.1	2.9
unrestricted	0.079	0 %	36.4	39.3	36.4	36.4	0.0	2.9
		50 %	13.4	39.3	15.5	36.4	-2.1	2.9
		-50 %	48.2	38.5	45.7	34.8	2.5	3.7
	0.106	0 %	25.9	38.5	25.8	34.8	0.1	3.7
		50 %	3.5	38.5	5.8	34.8	-2.3	3.7
		-50 %	71.1	35.7	70.2	33.2	0.9	2.5
	0.051	0 %	51.2	35.7	51.2	33.2	0.0	2.5
		50 %	30.7	35.7	32.1	33.2	-1.4	2.5
		-50 %	59.2	39.7	57.2	36.4	2.0	3.3
restricted	0.079	0 %	36.4	39.7	36.4	36.4	0.0	3.3
		50 %	13.6	39.7	15.5	36.4	-1.9	3.3
		-50 %	48.1	38.7	45.7	34.8	2.4	3.9
	0.106	0 %	25.9	38.7	25.8	34.8	0.1	3.9
		50 %	3.7	38.7	5.8	34.8	-2.1	3.9
		-50 %	72.0	36.6	70.2	33.2	1.8	3.4
	0.051	0 %	50.0	36.6	51.2	33.2	-1.2	3.4
		50 %	30.0	36.6	32.1	33.2	-2.1	3.4
		-50 %	59.5	40.4	57.2	36.4	2.3	4.0
fully committed	0.079	0 %	36.3	40.4	36.4	36.4	-0.1	4.0
		50 %	13.2	40.4	15.5	36.4	-2.3	4.0
		-50 %	48.3	39.1	45.7	34.8	2.6	4.3
	0.106	0 %	25.8	39.1	25.8	34.8	0.0	4.3
		50 %	3.4	39.2	5.8	34.8	-2.4	4.4

Table 5
Commitment vs. No Commitment: Allocation Differences for Competing Managers

This table presents the difference in fund allocations between a manager that does not invest any of her entire wealth to her fund but who can only invest in the opportunities available to the retail investor in her personal account, vs. a manager who invests all of her wealth to her fund. Panel A corresponds to the case of two managers with the same, constant, ability. The parameter values for Panel A are the same as in Figure 8. Panel B corresponds to the case of two managers with differential, constant, ability. The left subpanel corresponds to the manager with the higher level of ability. The parameter values for Panel B are the same as in Figure 9. Panel C corresponds to the case of a manager with constant ability vs. a manager with a mean-reverting level of ability. The left subpanel corresponds to the manager with the constant level of ability. The parameter values for Panel C are the same as in Figure 10.

Panel A

A/B	0.1%	0.5%	1.0%	5%	10%
0.1%	1%	1%	1%	3%	4%
0.5%	2%	2%	3%	4%	5%
1.0%	3%	4%	4%	5%	6%
5%	9%	9%	10%	11%	11%
10%	14%	14%	14%	15%	15%

Panel B

A/B	0.1%	0.5%	1.0%	5%	10%
0.1%	1%	2%	2%	5%	7%
0.5%	2%	2%	3%	6%	7%
1.0%	3%	3%	4%	6%	8%
5%	7%	7%	8%	10%	11%
10%	10%	10%	11%	12%	14%

A/B	0.1%	0.5%	1.0%	5%	10%
0.1%	3%	9%	16%	40%	52%
0.5%	3%	10%	16%	40%	52%
1.0%	4%	10%	17%	40%	53%
5%	6%	12%	19%	41%	53%
10%	9%	14%	20%	42%	54%

Panel C

A/B	0.1%	0.5%	1.0%	5%	10%
0.1%	0%	-2%	-4%	-8%	-4%
0.5%	2%	0%	-2%	-5%	-2%
1.0%	4%	3%	1%	-3%	1%
5%	11%	10%	8%	5%	8%
10%	15%	14%	13%	10%	12%

A/B	0.1%	0.5%	1.0%	5%	10%
0.1%	-2%	-11%	-24%	-46%	-22%
0.5%	-2%	-10%	-23%	-46%	-22%
1.0%	-1%	-10%	-22%	-45%	-22%
5%	0%	-7%	-19%	-43%	-21%
10%	2%	-6%	-16%	-41%	-20%