

Relative Pay for Non-Relative Performance: Keeping up with the Joneses with Optimal Contracts¹

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ABSTRACT. We consider a multi-agent contracting setting when agents derive utility based in part on their pay relative to their peers. Because agents' productivity is affected by common as well as idiosyncratic shocks, it is optimal to base pay on the agent's performance relative to a benchmark of his peers. But when agents have "keeping up with the Joneses" (KUJ) preferences and care about how their pay compares to that of others, relative performance evaluation also increases agents' perceived risk. We show that when a single principal (or social planner) can commit to a public contract, the optimal contract hedges the risk of the agent's relative wage without sacrificing efficiency. While output is unchanged, however, hedging makes the contracts *appear* inefficient in the sense that performance is inadequately benchmarked. We also show that when there are multiple principals, or the principal is unable to commit, efficiency is undermined. In particular, KUJ effects induce agents to be more productive, but average wages increase even more, reducing firm profits. We also show that if the principal cannot commit not to privately renegotiate contracts, then wages and effort are increased when KUJ effects are weak, but are reduced, enhancing efficiency, when KUJ effects are sufficiently strong. Finally, public disclosure of contracts across firms can cause output to collapse.

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1. Introduction

Optimal contracting and incentive theory has provided powerful insight into the optimal design of compensation contracts. Chief among them, for instance, is the idea that contracts should provide higher compensation when output suggests that the agent was more likely to have engaged in desired behavior. In particular, Holmstrom's (1992) Informativeness Principle states that any measure of performance that reveals information about the agent's effort should be included in the compensation contract. A prime example is the use of Relative Performance Evaluation (RPE), in which the agent's performance is measured relative to an average of her peers in order to filter out common sources of noise. In other words, optimal contracts should *not* "pay for luck" due to aggregate shocks, but only pay for indicators of individual performance.

The benefit of RPE is that it allows compensation to remain sensitive to the components of output that the agent controls, while reducing his exposure to aggregate fluctuations which he cannot control. Despite this clear benefit, in practice it is observed much less frequently than theory would predict.¹ In this paper we consider a possible explanation for lack of RPE in practice: that in addition to their absolute wage, agent's care about their wage *relative to* the wages of their peers. When agents have a "keeping up with the Joneses" (KUJ) component to their preferences, relative performance evaluation increases the agent's perceived risk. We derive optimal contracts in this context and show that the sensitivity of pay to aggregate performance benchmarks will depart dramatically from the predictions of RPE, but this departure need not entail a loss of efficiency.

Our model includes many agents who take hidden effort to produce output that is subject to both common and idiosyncratic shocks. Agents receive a compensation contract which specifies their wage as a function of their own output as well as the aggregate (or average) output of others. Agents are risk averse and have preferences that are increasing in both their own wage as well as

¹ For empirical evidence of "pay for luck" in the context of CEO compensation, see e.g. Murphy (1985), Coughlan and Schmidt (1985), Antle and Smith (1986), Gibbons and Murphy (1990), Janakiraman, Lambert, and Larker (1992), Garen (1994), Aggarwal and Samwick (1999a,b), Murphy (1999), Frydman and Jenter (2010), and Jenter and Kanaan (2014).

the difference between their own wage and the average wage of others. The relative sensitivity to absolute versus relative wages determines the strength of the KUJ effect in our model.

We begin in Section 2 by analyzing the RPE benchmark absent any KUJ effect, and show that as expected, compensation in the optimal contract is based on a measure of the agent's relative performance; that is, compensation is positively related to the agent's own output and negatively related to the output of others, in relative proportions that depend on the correlation between agents' output. We then compare this outcome with the outcomes obtained when agents care also about their relative wage across a variety of contracting settings.

In Section 3 we consider a setting in which a single principal contracts with multiple agents, with incentive terms publicly disclosed.² We show that the optimal contract is designed to hedge the agent's risk exposure that arises from relative wage concerns. By doing so, the effect of KUJ preferences on efficiency is neutralized so that average wages, output, and utility are identical to the standard RPE benchmark model. The observed wage sensitivities, however, are very different. In particular, we show that each agent's wage sensitivity to the output of others is increasing with the strength of the KUJ effect, and becomes positive if they are strong enough. Indeed, in the limit we find that agents are paid on the basis of total aggregate output, and the sensitivity to individual performance disappears. Thus, empirical measures of RPE would fail in this context – agents are paid for luck. Yet despite this divergence, we show that efficiency is maintained.³ In a sense, KUJ effects are neutralized by hedging the agent's risk in the optimal contract, so that optimal incentives are maintained. Overall we find that as KUJ concerns increase, wage volatility declines and the correlation between agents' wages increases.

Next we consider a setting with independent principal-agent pairs (for example, boards and CEOs). We show in Section 4 that in this case, KUJ preferences have the same impact on relative wage sensitivities as in the single principal case (and so again RPE will fail empirically). In contrast to the prior setting, however, an externality arises as each principal ignores the effect of his agent's compensation on the utility of agents at other firms. As a result, with separate

² Disclosure matters in our setting because agents' incentives also depend on the wages of others. We consider alternative disclosure regimes in Sections 5 and 6.

³ This result is similar to that of Bartling (2011), who also shows that RPE incentive schemes may no longer be optimal with KUJ preferences, though in his setting, in which agents care about the expected ex-ante inequality, this entails a loss of productive efficiency.

principles the overall power of incentives increases, raising both effort and output as relative wealth concerns increase. These high-powered incentives impose higher risk on agents, who must then be compensated with higher average wages. The net effect is a decrease in firm profits (net of wages). Thus, when principals are independent, the externality associated with KIJ preferences leads to pay for luck – which is efficient – combined with inefficiently high productivity and wages, leading to decreased profits.

In Section 5 we consider a team setting in which independent principals each manage multiple agents (e.g. competitors with multiple workers in similar jobs). We then consider equilibria when the principal cannot commit not to privately renegotiate with individual agents. Renegotiation reduces inefficiency by raising effort and incentives when KIJ preferences are weak, as the principal tries to raise output in order to lower his obligation to other agents. But when KIJ preferences are strong, this effect is reversed and effort is lowered, raising efficiency in equilibrium.

Finally, in Section 6 we consider a setting in which independent principals disclose their compensation contracts externally (for example as a result of executive compensation disclosure requirements). In that case, other agents may adjust their effort choices in response to the contracts they observe for others. We show that in this case, the relative weight that contracts put on aggregate performance increases (compared to a setting with non-disclosure). When KIJ effects are weak, incentives are inefficiently strong (but not as strong as with non-disclosure). However, incentives may collapse when KIJ effects are very strong.

1.1. Related Literature

There is extensive empirical literature that has for the most part rejected the RPE hypothesis that CEO compensation should depend on relative performance, and so be negatively related to the performance of peers. Examples include: Antle and Smith (1986), Barro and Barro (1990), Jensen and Murphy (1990), Janakiraman, Lambert and Lacker (1992), Hall and Liebman (1998), Joh (1999), Aggarwal and Samwick (1999a,b), and Garvey and Milbourn (2003) . Importantly, most of the evidence in these documents a *positive* relation between other firm's performance and CEO compensation, in direct contrast to the standard RPE prediction. Indeed, Bertrand and Mullainathan (2001) find that CEO pay responds as much to a lucky dollar as a general dollar.

While a number of hypotheses have been put forth to explain this empirical failure of RPE, as we demonstrate, Keeping up with the Joneses preferences delivers a simple intuitive resolution for these findings, and the positive dependence on peer performance.

While Keeping/Catching up with the Joneses and habit formation preferences have been used in asset pricing applications starting with Abel (1990), they have received much less attention in explaining behavior in the corporate finance domain. Ederer and Pataconi (2010) introduce status considerations into a tournament setting analyzing implications for the provision of incentives. Goel and Thakor (2010) use envy-based preferences for managers to explain merger waves. Dur and Glazer (2008) consider the optimal contract, with contractible effort, for an employee that is envious of his employer. Goel and Thakor (2005) consider within firm capital allocation decisions of division managers where each manager derives direct utility from wages, and in addition envies both the wages of other managers and their capital allocation as well. Their analysis focuses on induced capital distortions, ignoring the moral hazard and contracting considerations which are the focus of our analysis.

Closer to part of our analysis, Bartling and von Siemens (2010) consider the impact of envy on contracts in a general moral hazard model when a principle hires two agents that are envious of each other. They show that envy can have both cost-increasing and cost-decreasing effects for the principle, and argue that with risk-averse agents and without limited liability envy can only increase the costs of providing incentives. The scope of their analysis is limited by the fact that they do not derive explicit optimal contracts. Bartling (2011) analyzes a contracting setting with one principle hiring two agents, when contracts are observable. The two agents suffer a disutility associated with the ex-ante expected wage inequality; thus, even if one agent earns a higher wage ex-post, his utility is still reduced by the possibility that he could have earned a lower wage. Miglietta (2008) assumes risk aversion both with respect to absolute wage and inequality, and considers also the case with one principle and N agents. None of the above papers scale the agents' outside options to make appropriate welfare comparisons as preferences vary, and thus do not obtain our efficiency results. Our additional contributions compared to these papers are three-fold. First, we investigate when contract disclosure within teams is optimal and compare the associated optimal contracts. Second, we consider a market wide equilibrium with multiple principles, analyzing the contracting externalities across principles. In doing so, we also analyze

how contracts vary as the degree of contract transparency across firms varies. Third, we contrast contracts where the peer group comprises of agents within the firm to those where peers are employed by other firms.

Our explanation for why CEOs pay is increasing in peer firm output is distinct yet complementary to prior proposed explanations which we briefly discuss below.

Aggarwal and Samwick (1999b) abstract from managerial effort choice considerations and show that with publically observable contracts, serving in part as a commitment device, when firms are product market compliments compensation increases in industry performance. In their model optimal contracts are identified only up to the ratio between the own and rival pay-performance sensitivity, and not their levels. Our explanation focuses instead on the managerial effort channel, and does not rely on complementarity.

Gopalan, Milbourn and Song (2010) assume a key CEO role is to take advantage of future sector movements. Consequently, the optimal contract rewards the CEO for firm performance induced by sector movements so as to provide incentives to exert effort to forecast these movements and choose the firm's optimal exposure to them.

Garvey and Milbourn (2003) argue that the degree of RPE in compensation contracts will be increasing in the manager's private cost of hedging and decreasing in firms' cost of providing RPE. Our model assumes it is costless for the firm to implement contracts and assumes all managers' wealth is coming from the compensation they receive from the firm.

Himmelberg and Hubbard (2000) and Oyer (2004) argue that the positive dependence of compensation on peer performance results from the fact that the value of executives' outside opportunities are also market sensitive. While the two explanations are not mutually exclusive, evidence in Duchin, Goldberg, and Sosyura (2014) supports ours as an independent channel. They show that division managers' compensation depends positively on other divisions' performance, and that common membership in social clubs, shared alumni networks and joint board appointments among conglomerate's division managers amplifies the spillovers of compensation shocks across divisions.

In addition to proposing a new mechanism for explaining the positive dependence on peer performance, we derive novel cross sectional predictions regarding the degree of RPE. First, we

compare different environments contrasting predictions when one principle hires multiple agents to that with multiple principles each hiring an agent. Second, we contrast contracts under different degrees of transparency within and/or across firms. We also produce predictions linking the degree of competition (number of peer firms) and the compensation sensitivity to peer firms' performance.

We also contribute to the literature by showing that some typical comparative statics in the contracting environment are overturned when agents have relative wealth concerns. Aggarwal and Samwick (1999a) focus their empirical tests of the principle-agent model on the sensitivity of the ratio of the weight on industry performance to the weight on own firm performance to the beta relative to the industry. Our model demonstrates that when managers have keeping up with the Joneses preferences, some of the predictions linking, for example, output volatility to the use of relative performance measures differ from those of standard models.

2. Basic Model

We consider a setting with $n+1$ total agents. We make the standard assumption that the utility of each agent i is increasing in his own wage, w_i , and decreasing in his effort, a_i . We depart from the usual principal-agent framework, however, by assuming that agents care about their wage relative to that of their peers. In particular, to capture this effect, we assume the utility of agent i decreases with the average wage of his peers, denoted by

$$w_{-i} \equiv \frac{1}{n} \sum_{j \neq i} w_j. \quad (1)$$

For tractability and to avoid wealth effects, we assume agents have CARA utility and denominate disutilities in units of consumption. Specifically, let $u(c) = -e^{-2\lambda c}$ and define the agent's utility as

$$U(w_i, w_{-i}, a_i) \equiv u\left(\frac{w_i - \delta w_{-i}}{1 - \delta} - \psi(a_i)\right). \quad (2)$$

Here ψ is the disutility of effort and $\delta < 1$ captures the strength of the relative wealth effect. We interpret $\frac{w_i - \delta w_{-i}}{1 - \delta}$ as the agent's "relative wage." This formulation is equivalent to specifying the agent's effective wage as

$$w_i + \hat{\delta}(w_i - w_{-i})$$

with $\hat{\delta} = \delta / (1 - \delta)$.⁴

We assume a quadratic disutility of effort with $\psi(a) = a^2 / (2k)$, so that the parameter $k > 0$ indexes the private cost associated with effort. Finally, we refer to

$$c_i \equiv \frac{w_i - \delta w_{-i}}{1 - \delta} - \psi(a_i) \quad (3)$$

as the agent's "adjusted consumption."

We consider a simple production technology with additive shocks. Specifically, the output q_i of agent i is equal to a constant plus effort plus noise:⁵

$$q_i \equiv q_0 + a_i + \dot{q}_i. \quad (4)$$

The random shocks \dot{q}_i are joint normal with mean zero and variance σ^2 , and have a pairwise correlation of $\rho \in [0, 1)$.⁶ Without loss of generality, we let $\sigma = 1$ by simply rescaling output (we could alternatively normalize the level of risk aversion λ). Adopting the same notation as we did with wages, we write q_{-i} to denote the average output of the agent's peers, \dot{q}_{-i} to denote their average shock, etc.

Note that in this setting, the first-best effort level maximizes $a_i - \psi(a_i)$ and thus $a_i = k$.

However, effort choices are hidden and subject to moral hazard. Appropriate compensation

⁴ What is critical in both cases is that, to allow for meaningful comparative statics, we have normalized the relative wage so that the importance of the wage component of consumption compared with the effort cost remains constant as we consider alternative values for δ . Also, while it is not our main focus, we can allow $\delta < 0$ to consider altruistic preferences.

⁵ We can interpret the constant q_0 as corresponding to output that the agent generates which can be easily monitored and so not subject to an agency problem.

⁶ Equivalently, we can write the shocks as $\dot{q} = \sqrt{1 - \rho}\eta_i + \sqrt{\rho}\eta_C$, with η independent standard normal.

contracts are needed to motivate the agent. We restrict attention to linear compensation contracts of the form:⁷

$$w_i = m_i + x_i q_i + y_i \sum_{j \neq i} q_j = m_i + x_i q_i + n y_i q_{-i} \quad (5)$$

where m_i is a constant, x_i is the sensitivity of the agent's wage to his own output, and y_i is the sensitivity of his wage to the aggregate output of his peers. Equivalently, $n y_i$ is the sensitivity to the *average* output of other agents.

2.1. Relative Performance Evaluation

Before we begin, it is useful to consider the role of relative performance evaluation (RPE) in this context. When the correlation ρ between the agents' shocks is positive, there is a common component to output, and thus the output of other agents will be informative with regard to agent i 's shock. In particular, given the average shock $\bar{\omega}_{-i}$ of the agent's peers, we have

$$E[\bar{\omega}_i | \bar{\omega}_{-i}] = \frac{n\rho}{1-\rho+n\rho} \bar{\omega}_{-i}. \quad (6)$$

Therefore, in a standard moral hazard setting ignoring relative wealth concerns, the optimal signal (up to a constant) upon which to base the agent's compensation is

$$q_i - \theta_{n,\rho} q_{-i}, \quad (7)$$

where we define the RPE benchmark

$$\theta_{n,\rho} \equiv \frac{n\rho}{1-\rho+n\rho}. \quad (8)$$

This signal minimizes the residual risk imposed upon the agent, which is given by

$$\text{Var}(q_i - \theta_{n,\rho} q_{-i}) = \sigma_{n,\rho}^2 \equiv \frac{(1-\rho)(1+n\rho)}{1-\rho+n\rho}. \quad (9)$$

⁷ Goukasian and Wan (2010) demonstrate optimality of linear contracts in this setting in a continuous time context as in Holmstrom and Milgrom (1987). While there is a typographical error in some of their reported results, similar findings apply here, and so under their assumptions our use of linear contracts is without loss of generality.

Note that when $\rho > 0$ and n becomes large, $\theta_{n,\rho} \rightarrow 1$ and $\sigma_{n,\rho}^2 \rightarrow 1 - \rho$, in which case the common risk factor is perfectly filtered out.

Thus, with the contracts in (5), the standard RPE outcome would predict

$$\frac{ny_i}{x_i} = -\theta_{n,\rho} \quad (10)$$

These results form the basis for standard tests of RPE in the empirical literature, which generally conclude that compensation tends to be much less sensitive to peer performance than is predicted by an optimal contracting framework, and indeed often has the opposite sign – pay is positively related to aggregate performance. One of the key goals of our paper is to understand how “pay for luck” can emerge when agents have relative wealth concerns, and how it may effect productivity and profits.

2.2. Effort and Payoffs

Because effort is hidden and independently chosen, each agent will chose his own effort taking as given his own wage contract as well as the wage contracts and effort choices of others. As in the standard principal agent model, the agent’s own effort affects his utility directly via the disutility of effort and the sensitivity of his wage to his own output. Relative wealth concerns add yet a third channel, however, as illustrated in Figure 1: By raising aggregate output, the agent’s own effort affects the benchmark, and thus the wage, of his peers, which ultimately determines the agent’s perception of his own wage.

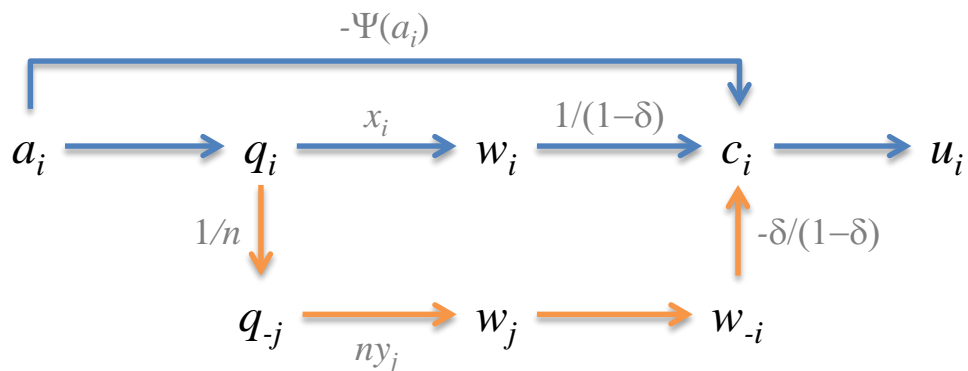


Figure 1: Alternative Channels by which Effort Impacts Utility

The agent's optimal effort choice will depend on the magnitude of each of these channels, as shown in the following lemma.

LEMMA 1. Given wage contracts (m, x, y) , agent i chooses effort $a_i = k\alpha_i$ where

$$\alpha_i \equiv \frac{x_i - \delta y_{-i}}{1 - \delta} \quad (11)$$

is the sensitivity of the agent's relative wage to his own effort. Effort is below first best if $\alpha_i < 1$.

PROOF: Observe that $\partial w_i / \partial a_i = x_i$ and

$$\begin{aligned} \frac{\partial w_{-i}}{\partial a_i} &= \frac{1}{n} \sum_{j \neq i} \frac{\partial}{\partial a_i} (m_j + x_j q_j + y_j q_{-j}) \\ &= \frac{1}{n} \sum_{j \neq i} y_j n \frac{1}{n} \sum_{k \neq j} \partial q_k / \partial a_i \\ &= y_{-i} \end{aligned} \quad (12)$$

Therefore, taking the wage contracts and actions of others as given, the agent will choose effort a_i to maximize his utility, which has the first order condition:

$$\frac{\partial}{\partial a_i} E[U(w_i, a_i, w_{-i})] = E[u'(c_i)] \left(\frac{x_i - \delta y_{-i}}{1 - \delta} - \frac{1}{k} a_i \right) = 0,$$

where c_i is the agent's adjusted consumption. Solving for a_i yields the result. ■

Again, because of the agent's relative wealth concerns, his own effort will depend on the sensitivity of other agents' wages to his realized output. If $y_{-i} < 0$, so that agents are penalized if others perform well, then relative wealth concerns will strengthen the agent's overall incentives.

To evaluate payoffs, note that with normally distributed consumption and CARA utility agents will have mean-variance preferences. That is, given consumption $c \sim N(\mu, \sigma^2)$, we can evaluate the agent's utility in terms of the corresponding certainty equivalent consumption level

$$u^{-1}(E[u(c)]) = \mu - \lambda \sigma^2.$$

In general, each agent's payoff will depend upon all other contracts, as these will determine the distribution of the average peer wage. Because agents are ex ante identical in our model, we expect that equilibrium wage contracts, and thus actions, will be symmetric. That said, equilibrium incentives may depend upon the payoffs that would be obtained were the agent to receive a different contract. As a result, it is useful to evaluate the payoff for agent i when all other agents have an identical contract (and choose the same action) but this contract may differ from that of agent i . Specifically, suppose

$$(m_j, x_j, y_j, a_j) = (m_{-i}, x_{-i}, y_{-i}, a_{-i}) \text{ for all } j \neq i.$$

Then we have the following characterization.

LEMMA 2. Suppose agents $j \neq i$ have symmetric contracts. Then expected wages are given by

$$E[w_i] = m_i + x_i a_i + y_i n a_j$$

$$E[w_{-i}] = E[w_j] = m_j + x_j a_j + y_j (a_i + (n-1)a_j)$$

Letting $\alpha_i \equiv \frac{x_i - \delta y_j}{1 - \delta}$ be the agent's total exposure to his own output and

$\beta_i \equiv \frac{n y_i - \delta x_j - (n-1) \delta y_j}{1 - \delta}$ be the agent's total exposure to the output of others, agent i 's

adjusted consumption c_i has mean and variance

$$E[c_i] = \frac{m_i - \delta m_j}{1 - \delta} + \alpha_i a_i + \beta_i a_j - \psi(a_i)$$

$$\text{Var}(c_i) = \left(\alpha_i^2 + \frac{1}{n} \beta_i^2 \right) (1 - \rho) + (\alpha_i + \beta_i)^2 \rho$$

PROOF: Expected wages follow by direct calculation given the contract and production technology. The sensitivity α_i to the agent's own shock follows as in the previous lemma. The agent's sensitivity to the average shock of others is given by

$$(1 - \delta)^{-1} \left(\underbrace{ny_i}_{\text{from } w_i} - \delta \underbrace{(x_j + (n-1)y_j)}_{\text{from } w_{-i}} \right) = \beta_i .$$

The result then follows since the average idiosyncratic shock of others has $1/n$ times the variance of an individual idiosyncratic shock, and the total exposure to the common shock is $\alpha_i + \beta_i$. ■

The preceding lemma allows us to recast the contracting problem to a choice of the parameters (α_i, β_i) which determine the agent's exposure to his own risk and to the common risk. Because the agent i 's incentives are determined solely by α_i , it is optimal to choose β_i to minimize the risk of the agent's adjusted consumption. The following result characterizes the minimum variance contract, and relates it to the standard RPE solution discussed earlier.

LEMMA 3. Given α_i , the variance of the agent's adjusted consumption c_i is minimized with

$$\beta_i = -\alpha_i \left(\frac{n\rho}{1-\rho+n\rho} \right) = -\alpha_i \theta_{n,\rho} \quad (13)$$

In that case, $\text{Var}(c_i) = \alpha_i^2 \sigma_{n,\rho}^2$.

PROOF: To minimize

$$\text{Var}(c_i) = \left(\alpha_i^2 + \frac{1}{n} \beta_i^2 \right) (1-\rho) + (\alpha_i + \beta_i)^2 \rho,$$

we can solve for β_i from the first order condition

$$\frac{2}{n} \beta_i (1-\rho) + 2(\alpha_i + \beta_i) \rho = 0$$

verifying (13). Given the solution to β_i , we have

$$\begin{aligned}
\text{Var}(c_i) &= \alpha^2 \left(1 - \rho + \frac{1}{n} \left(\frac{\beta}{\alpha} \right)^2 (1 - \rho) + \left(1 + \frac{\beta}{\alpha} \right)^2 \rho \right) \\
&= \alpha^2 \left(1 - \rho + \frac{1}{n} \left(\frac{n\rho}{1 - \rho + n\rho} \right)^2 (1 - \rho) + \left(\frac{1 - \rho}{1 - \rho + n\rho} \right)^2 \rho \right) \\
&= \alpha^2 \left((1 - \rho) + \rho(1 - \rho) \frac{n\rho + (1 - \rho)}{(1 - \rho + n\rho)^2} \right) \\
&= \alpha^2 \left(\frac{(1 + n\rho)(1 - \rho)}{(1 - \rho + n\rho)} \right).
\end{aligned}$$

■

3. Single Principal, Public Contracts

We consider first a setting in which there is a single principal committing to a public contract for a set of $n+1$ agents. Because of the correlation in output, the principal obtains more precise information about each agent's effort by considering his output relative to that of his peers. In addition, the principal understands the agents' concerns regarding relative pay, and must consider this effect when determining how best to provide incentives.

The principal seeks the contract that will maximize the expected aggregate output of the agents net of the wages paid. Agents choose effort based on the contracts' incentives, and wages must be set to satisfy a participation constraint. Specifically, the timing is as follows:



Figure 2: Single Principal with Public Contracts

Thus, the principal solves the following optimization problem:

$$\begin{aligned}
& \max_{(m,x,y,a)} E\left[\sum_i q_i - w_i\right] \\
& \text{s.t. for all } i, \\
& \quad a_i = k\left(\frac{x_i - \delta y_{-i}}{1 - \delta}\right) \quad (IC) \\
& \quad E[c_i] - \lambda \text{Var}(c_i) \geq c_0 \quad (PC)
\end{aligned} \tag{14}$$

Because the fixed component of the agent's wage, m_i , can be reduced so that (PC) always binds, at the solution to the principal's problem we must have

$$E[c_i] = E\left[\frac{w_i - \delta w_{-i}}{1 - \delta}\right] - \psi(a_i) = c_0 + \lambda \text{Var}(c_i) \tag{15}$$

Next, note that $\sum_i w_i = \sum_i w_{-i}$, and therefore the expected aggregate wage bill satisfies

$$E\left[\sum_i w_i\right] = \sum_i c_0 + \psi(a_i) + \lambda \text{Var}(c_i) \tag{16}$$

As a result, we can reduce the principal's problem to⁸

$$\max_{(x,y,a)} \sum_i a_i - \psi(a_i) - \lambda \text{Var}(c_i) \quad \text{s.t. } (IC) \tag{17}$$

Note that (17) is the same problem faced by a social planner attempting to maximize total welfare, which consists of expected aggregate output net of the costs of effort and risk-bearing.

Given the convexity of both the effort cost and the variance of consumption in the parameters (x, y, a) , it is straightforward to show that the solution to (17) will be symmetric, so that the contracts will be identical for each agent. We characterize the optimal contract below.

⁸ Note that we have dropped the constant term $q_0 - c_0$ from the principal's objective. This baseline level of surplus is relevant if we consider the principal's participation constraint, which is to earn a non-negative profit. As long as this baseline surplus is nonnegative, the principal participation constraint will not bind in this case (since he can always achieve at least this level by paying a constant wage).

PROPOSITION I. Given a single principal who can commit to a public contract, the optimal contract is symmetric with $\beta = -\alpha\theta_{n,\rho}$ and

$$\alpha = \frac{1}{1 + 2\lambda k^{-1}\sigma_{n,\rho}^2} < 1. \quad (18)$$

Effort is given by $a = k\alpha$, and effort, expected wages, and profits are independent of δ .

The observed contract parameters are

$$x = \alpha - \delta(\alpha - y) = \alpha \left(1 - \delta \left(\frac{n + \theta_{n,\rho}}{n + \delta} \right) \right), \quad y = -\alpha \left(\frac{\theta_{n,\rho} - \delta}{n + \delta} \right). \quad (19)$$

PROOF: The (IC) constraint is simply $a_i = k\alpha_i$. Using the result of **LEMMA 3**, we can choose β_i to minimize variance and thus the optimal contract maximizes

$$k\alpha_i - \psi(k\alpha_i) - \lambda\alpha_i^2\sigma_{n,\rho}^2$$

The first order condition is

$$k - k\alpha_i - 2\lambda\alpha_i\sigma_{n,\rho}^2 = 0.$$

Hence the optimal solution is

$$\alpha_i = \frac{1}{1 + 2\lambda k^{-1}\sigma_{n,\rho}^2},$$

with $\beta_i = -\alpha_i\theta_{n,\rho}$ from **LEMMA 3**. From **LEMMA 1**, $a_i = k\alpha_i$. Given symmetry and the definitions in **LEMMA 2**, $\alpha_i = \alpha = \frac{x - \delta y}{1 - \delta}$ and so $x = \alpha - \delta(\alpha - y)$. Finally, we have

$$\beta_i = \beta = -\alpha\theta_{n,\rho} = \frac{(n - (n-1)\delta)y - \delta x}{1 - \delta} = \frac{(n(1 - \delta) + \delta)y - \delta(\alpha - \delta(\alpha - y))}{1 - \delta}$$

which we can solve for y as $y = -\alpha \left(\frac{\theta_{n,\rho} - \delta}{n + \delta} \right)$. ■

The results of **PROPOSITION I** are striking. In particular, (18) implies that the agent's effort choice and the optimal sensitivities (α, β) are independent of the strength δ of his relative wealth concerns. As a result, expected wages and output are independent of δ and thus there is no loss

(or gain) of efficiency induced by these preferences. Instead, the principal is able to undo the effect of these preferences through the contract itself. But, while the real outcomes are unaffected by δ , the contract sensitivities (x, y) are affected due to the implicit hedging of relative wealth effects embedded in the optimal contract. As relative wealth concerns increase, optimal contracts put more weight on the aggregate benchmark and less weight on the agent's own performance.

COROLLARY I.A For $\rho \in (0,1)$, as δ increase from 0 to 1,

- x decreases from α to $\alpha(1-\theta_{n,\rho})/(n+1)$,
- ny increases from $-\alpha\theta_{n,\rho} < 0$ to $n\alpha(1-\theta_{n,\rho})/(n+1) > 0$,
- The relative sensitivity y/x increases from $-\theta_{n,\rho}/n$ to 1.

PROOF: Immediate from **PROPOSITION I**. ■

The above results have important implications for empirical tests of RPE in the presence of relative wealth concerns. Absent these concerns, optimal signal extraction suggests that the agent's sensitivity to peer performance relative to his own should equal $-\theta_{n,\rho}$. With relative wealth concerns, the relative sensitivity to peer performance increases with δ . For $\delta > \theta_{n,\rho}$, the agent's wage will be *increasing* with the performance of his peers. Indeed, for δ close to 1, the relative sensitivity approaches 1 and thus the agent's wage will become proportional to *aggregate* output.

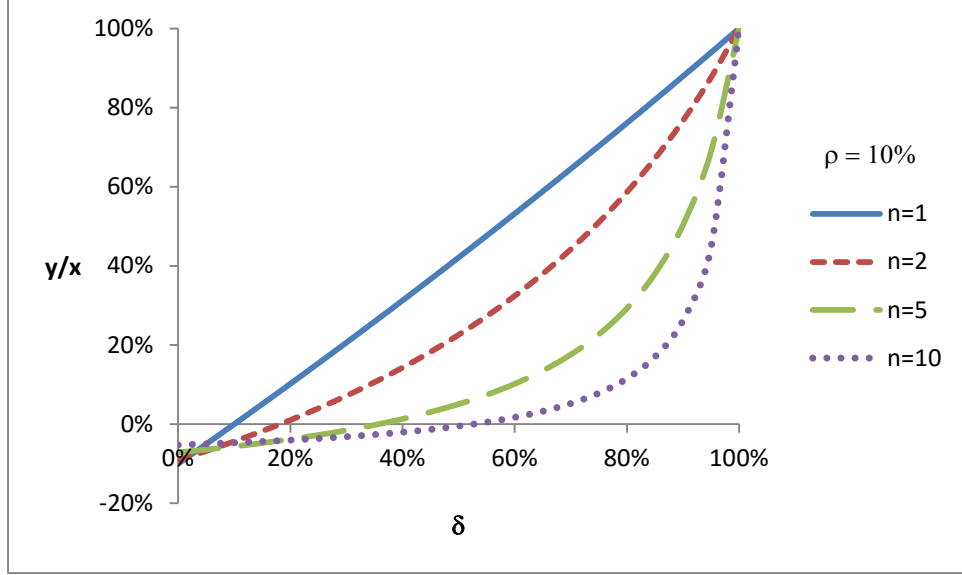


Figure 3: Relative Contract Sensitivity for Different Degrees of Relative Wealth Concerns

An additional empirical implication of our results is that when δ approaches one and wages become proportional to aggregate output, the dispersion between agents' wages will decline. Indeed, as we show below, the correlation between the wages of any pair of agents approaches one.

COROLLARY I.B For $\rho \in [0,1)$, as δ increase from 0 to 1,

- The volatility of each individual's wage w_i declines,
- The volatility of the agent's relative wage $w_i - w_{-i}$ declines to zero,
- The pairwise correlation between wages approaches 1.

PROOF: First,

$$\text{Var}(w_i) = (x + ny)^2 \rho + (x^2 + ny^2)(1 - \rho),$$

where the first term captures i 's wage exposure to the common shock and the second term his exposure to idiosyncratic shocks. Using the solution from **PROPOSITION I** we can calculate

$$\frac{\partial}{\partial \delta}(x + ny) = 0 \quad \text{and} \quad \frac{\partial}{\partial \delta}(x^2 + ny^2) = 2x'(x - y) < 0.$$

Next,

$$\text{Var}(w_i - w_{-i}) = (1 + \frac{1}{n})(x - y)^2(1 - \rho)$$

which declines to zero by Corollary I.A. Finally,

$$\text{Cov}(w_i, w_j) = (x + ny)^2 \rho + (2xy + (n - 1)y^2)(1 - \rho)$$

which implies

$$\text{Corr}(w_i, w_j) = 1 - (x - y)^2(1 - \rho) / \text{Var}(w_i).$$

■

We illustrate this result in Figure 4.

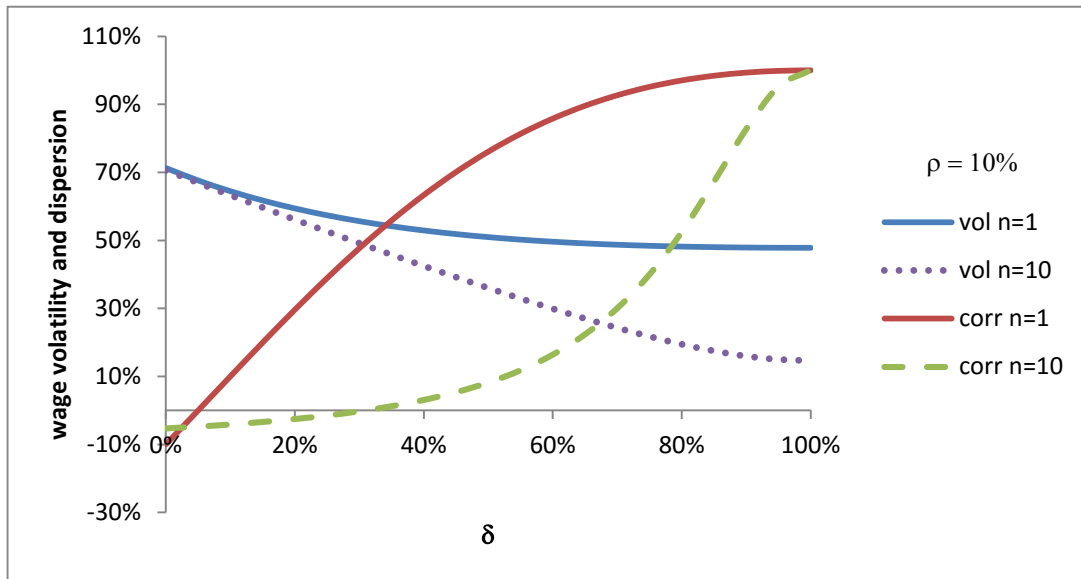


Figure 4: Wage Volatility Declines and Correlation Increases with Relative Wealth Concerns

Finally, we derive the following explicit comparative statics from our model. An increase in risk aversion or volatility decreases efficiency (as usual), but does not change the contract's relative sensitivity to own performance versus the benchmark. Also, while the weight on own performance increases with correlation in the standard model, it may decrease in the presence of relative wealth concerns. Similarly, while the relative wage sensitivity ny/x decreases with n in the standard RPE framework, the reverse may be true here.

COROLLARY I.C We have the following comparative statics:

- As risk version λ (or volatility) increases, effort and x decrease, while y/x remains constant.⁹

- As correlation ρ increases and aggregate output becomes more informative, effort increases, while y and y/x decrease. Finally, x is decreasing if

$$\delta \geq \frac{n^2 + n}{n^2 + n + k/(2\lambda)}, \text{ increasing when } \delta = 0, \text{ and otherwise is u-shaped.}$$

- As the number of agents n increases (which also increase the informativeness of aggregate output), if $\rho = 0$, then ny/x is increasing (or constant if $\delta = 0$). If

$$\rho > \frac{\delta^2}{\delta + 2(1 - \delta^2)}, \text{ then } ny/x \text{ is decreasing with } n. \text{ Otherwise, it is tent-shaped.}^{10}$$

Effort increases with n unless $\delta = \rho = 0$, in which case it is constant.

PROOF: See Appendix. ■

PROPOSITION I and the results highlighted above provide an important “irrelevance” benchmark for contractual settings with “keeping up with the Joneses”-type preferences. Indeed, our basic efficiency result is actually far more general than our specific setting, as the following result reveals:

⁹ With output volatility σ^2 , instead of normalized to 1, all results follow by replacing λ with $\sigma^2\lambda$. Thus, comparative statics with respect to output volatility are identical to those with respect to λ .

¹⁰ In the special case $\rho = \frac{\delta^2}{\delta + 2(1 - \delta^2)} > 0$, then ny/x is constant for $n = 1, 2$ and then decreases.

PROPOSITION II. Consider any utility functions u_i , distribution of shocks \dot{q} , effort costs Ψ_i , and let the wage for each agent i be an arbitrary function w_i of the vector of outputs q . Then aggregate effort and expected output are independent of δ , and the optimal contract satisfies

$$w_i = (1-\delta)w_{0i} + \delta w_{-i}, \quad (20)$$

where w_{0i} is the optimal contract for i when $\delta = 0$. In the specialized setting of

PROPOSITION I, we have

$$x = (1-\delta)x_0 + \delta y \text{ and } ny = (1-\delta)ny_0 + \delta(x + (n-1)y). \quad (21)$$

where (x_0, y_0) are the optimal contract sensitivities when $\delta = 0$.

PROOF: We begin by verifying the result in the context of **PROPOSITION I**. The case for x is immediate. For y , note that

$$\begin{aligned} ny &= (1-\delta)ny_0 + \delta(x + (n-1)y) \\ \Leftrightarrow (1-\delta)ny &= (1-\delta)ny_0 + \delta(x - y) \\ \Leftrightarrow ny &= ny_0 + \delta(x - y) / (1-\delta) = ny_0 + \delta(\alpha - y) = -\alpha \left(\theta_{n,\rho} - \delta \frac{n + \theta_{n,\rho}}{n + \delta} \right) = -n\alpha \left(\frac{\theta_{n,\rho} - \delta}{n + \delta} \right). \end{aligned}$$

To understand the more general result, note that (20) implies that given contract w_i , agent i 's relative wage is equal to w_{0i} . Thus, agents' incentives with contracts f_i and $\delta > 0$ are identical to their incentives with contracts w_{0i} and $\delta = 0$. Moreover, summing (20) over all agents and using the fact that $\sum_i w_i = \sum_i w_{-i}$, we see that aggregate wages are identical. Thus, the principal can provide the same incentives at the same cost for any δ . ■

As **PROPOSITION II** highlights, efficiency follows from two key aspects of our model. First, the space of wage contracts must be sufficiently rich so that (20) is feasible; in our setting with linear contracts, symmetry across agents allows us to write the contract as a function of only the agent's own output and the aggregate output of others. Second, the impact of relative wage dispersion on utility is linear in consumption; if alternatively agents were more or less risk averse about their relative wage than about their absolute wage, changing δ would change their overall

risk aversion and therefore necessarily impact efficiency. But with a sufficiently rich contract space, as long as relative wealth concerns do not change effective risk aversion or the relative cost of effort, efficiency is unaffected. What is affected is the form of the optimal contract, with a significant departure away from the standard prediction of relative performance evaluation toward compensation based on aggregate performance.

4. Many Principals

We now consider a setting in which there are many independent principals. We consider first the case in which each principal manages a single agent, but the agent's performance can be benchmarked against the performance of agents at other firms. Such a setting could correspond, for example, to the case of CEOs within an industry: CEO compensation is set independently by firm boards, but because firms may be affected by common shocks, performance measures are often benchmarked to industry averages. At the same time, CEOs may evaluate their wage relative to those of their peers.¹¹

With multiple principals, an externality arises in that each principal does not account for the negative impact of the wage he pays on the utility of agents at other firms. As a result, equilibrium effort and productivity increase relative to the single principal case, and may even exceed the first best. But this increase in output comes at the expense of excessive wages, causing profits and efficiency to decline. Interestingly, despite these changes, our results from Section 3 regarding relative performance evaluation remain unchanged.

In Section 4.2, we generalize our results to principals who manage multiple agents, and show that the effects outlined above are dampened as the principal internalizes the impact of higher wages throughout the organization.

4.1. Single Agent Contracts

Consider first the setting in which each principal contracts privately and independently with a single agent. We assume the following timing:

¹¹ Alternatively, our setting with individual principals might even apply within a firm, if agents are overseen by different managers and these managers set contracts in an uncoordinated fashion.

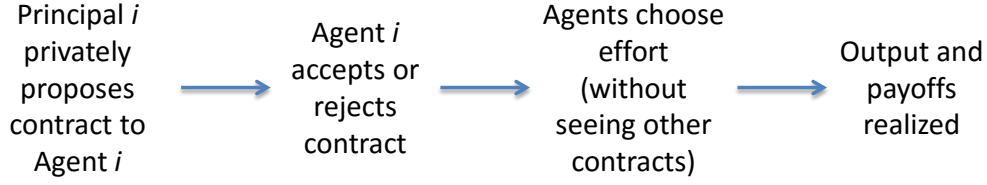


Figure 5: Many Independent Principals Setting Contracts Privately

When individual contracts are private, principal-agent pair i will negotiate taking *as given* the equilibrium contracts and action choices of others. Of course, in equilibrium these expectations should be correct. Given $(m_{-i}, x_{-i}, y_{-i}, a_{-i})$, the optimal contract for agent i solves

$$\begin{aligned}
 & \max_{(m_i, x_i, y_i, a_i)} E[q_i - w_i] \\
 & \text{s.t. } a_i = k \left(\frac{x_i - \delta y_{-i}}{1 - \delta} \right) \quad (IC_i) \\
 & E[c_i] - \lambda \text{Var}(c_i) \geq c_0 \quad (PC_i)
 \end{aligned} \tag{22}$$

As in the prior setting, because the fixed component of the agent's wage, m_i , can be reduced so that (PC_i) always binds, (15) holds, which implies

$$E[w_i] = \delta E[w_{-i}] + (1 - \delta)[c_0 + \psi(a_i) + \lambda \text{Var}(c_i)] \tag{23}$$

As a result, we can reduce the principal's problem to

$$\max_{(x_i, y_i, a_i)} a_i - \left(\delta E[w_{-i}] + (1 - \delta)[\psi(a_i) + \lambda \text{Var}(c_i)] \right) \quad \text{s.t. } (IC_i) \tag{24}$$

Comparing (24) with the optimization for a single principal in (17), we can see that they coincide when $\delta = 0$. When $\delta > 0$, independent principals do not account for the negative externality of a higher wage for their own agent on the utility of other agents. This effect manifests itself in (24) as a lower weight on the cost of effort of inducing effort. In addition, each principal benefits from inducing actions that, by manipulating the performance benchmark, reduce the expected wage of other agents and thereby raises the utility of their own agent.

Of course, because agents are not fooled in equilibrium, these manipulations will not be effective – if agents anticipate that other agents will work harder and earn more, their wage will also need to be higher, and all wages will rise to the detriment of total overall welfare. In other words,

because each agent's wage imposes a negative externality on others, an inefficiency arises when contracts are set independently.

We derive the optimal solution below, where we use the superscripts “S” and “M” to denote the corresponding solutions from the single and multiple principal cases, respectively.

PROPOSITION III. Given independent principal-agent pairs who contract privately, the equilibrium contract is symmetric and has

$$(\alpha^m, \beta^m, x^m, y^m, a^m) = \frac{1}{1 - \delta(1 - y^s)} (\alpha^s, \beta^s, x^s, y^s, a^s). \quad (25)$$

As a result, equilibrium effort is above the single principal case when $\delta > 0$, and increases with δ . Welfare is decreasing in δ .

PROOF: Substituting $a_i = k\alpha_i$ from the (IC) constraint, using the result of **LEMMA 3** to set $\beta_i^m = -\alpha_i^m \theta_{n,p}$ to minimize variance, and finally using the fact that $\partial w_{-i} / \partial a_i = y_{-i}$ from (12), the optimization in (24) is equivalent to

$$\max_{\alpha_i} k\alpha_i - \delta y_{-i}(k\alpha_i) - (1 - \delta) \left(\psi(k\alpha_i) + \lambda \alpha_i^2 \sigma_{n,p}^2 \right)$$

The first order condition is

$$(1 - \delta y_{-i})k - (1 - \delta) \left(k\alpha_i + 2\lambda \alpha_i \sigma_{n,p}^2 \right) = 0,$$

and so the optimal solution is

$$\alpha_i^m = \frac{1 - \delta y_{-i}}{1 - \delta} \frac{1}{1 + 2\lambda k^{-1} \sigma_{n,p}^2} = \frac{1 - \delta y_{-i}}{1 - \delta} \alpha^s.$$

Imposing symmetry (which we show in the appendix is the unique equilibrium), we know that the mapping from (α^m, β^m) to (x^m, y^m, a^m) is unchanged from the single principal case. Thus,

$$y^m = -\alpha^m \left(\frac{\theta_{n,p} - \delta}{n + \delta} \right) = - \left(\frac{1 - \delta y^m}{1 - \delta} \alpha^s \right) \left(\frac{\theta_{n,p} - \delta}{n + \delta} \right) = \left(\frac{1 - \delta y^m}{1 - \delta} \right) y^s.$$

We can solve for y^m as

$$y^m = \frac{1}{1 - \delta(1 - y^s)} y^s,$$

and it is easy to see that this same scaling factor will apply to each of the contract variables.

Next, because $\alpha^s < 1$ and $\theta_{n,\rho} \in [0,1]$, we have

$$1 > y^s = -\alpha^s \left(\frac{\theta_{n,\rho} - \delta}{n + \delta} \right) > -\frac{1 - \delta}{n + \delta} > -\frac{1 - \delta}{\delta}, \quad (26)$$

so that the scaling factor exceeds 1 for $\delta > 0$. For the comparative statics with respect to δ , note that

$$\begin{aligned} \delta(1 - y^s) &= \delta \left(1 + \alpha^s \left(\frac{\theta_{n,\rho} - \delta}{n + \delta} \right) \right) = \delta \left(1 + \alpha^s \left(-1 + \frac{n + \theta_{n,\rho}}{n + \delta} \right) \right) \\ &= \delta(1 - \alpha^s) + \alpha^s (n + \theta_{n,\rho}) \left(\frac{\delta}{n + \delta} \right) \end{aligned} \quad (27)$$

which is strictly increasing in δ . Finally, because $E[w_i] = E[w_{-i}]$ in equilibrium, aggregate welfare is as in (17), and so declines as α^m departs from α^s . ■

As **PROPOSITION III** demonstrates, when contracts are determined independently, both effort and incentives will be distorted upward. The representation of the equilibrium contract in (25) is remarkably simple: The term $\delta(1 - y^s)$ elegantly captures both the negative externality of the agent's wage on others (via δ) and the desire to manipulate the benchmark (via y^s). Note that the two effects work in opposing directions when δ is high enough so that $y^s > 0$, but nevertheless the proposition shows that the delta effect always dominates.

Overall, independent contracting increases incentives and effort, but exposes the agent to increased risk. The higher disutility from risk leads to an overall reduction in welfare. Holding fixed agents' outside option, this setting would therefore show higher productivity and wages, but lower firm profitability, than the social planner solution in **PROPOSITION I**. Moreover, these

distortions increase with the degree of “keeping up with the Joneses” concerns on the part of agents. At the extreme, when δ is close to 1, effort will exceed the first best.¹² See Figure 6.

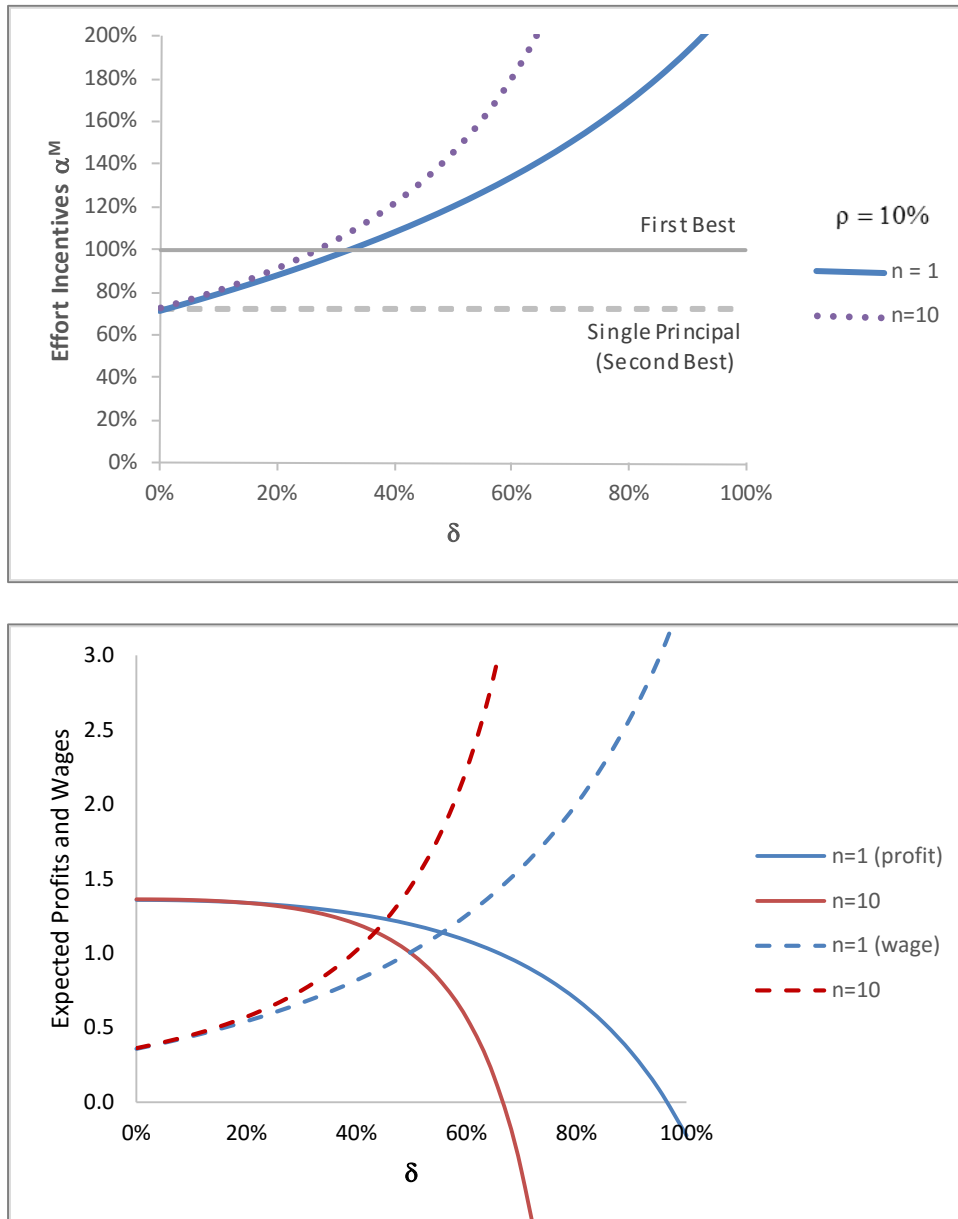


Figure 6: Effort Incentives, Profits and Wages with Multiple Principals

While effort and wages are distorted, however, because both x and y are simply rescaled: the *relative* sensitivity of the agent’s compensation to own versus others output is unchanged.

¹² When $\delta = 1$, effort incentives become $\alpha^m = \alpha^s / y^s = (n+1) / (1-\theta_{n,\rho}) > 1$.

COROLLARY III.A The relative sensitivity $y^m / x^m = y^s / x^s$.

PROOF: Immediate from **PROPOSITION III**. ■

Together, the results of Section 3 together with **PROPOSITION III** demonstrate the potential separation of efficiency from relative performance evaluation in the presence of relative wealth concerns. In Section 3 we showed that we can observe large deviations from the standard RPE contract while maintaining efficiency, while here we have shown that those same deviations of the contract can be associated with large inefficiency in the outcome.

Finally, we have the following comparative statics results:

COROLLARY III.B

- As λ increases, effort and x^m decrease, while y^m / x^m is unchanged.
- As ρ increases, effort increases, y^m decreases, and x^m initially decreases (if $\delta > 0$) and then increases.
- As n increases, if $\rho = \delta = 0$ effort is constant. If $\rho = 0$ or $\frac{k}{2\lambda} \leq \frac{1-\rho}{\delta}$ effort increases, otherwise, if $\frac{k}{2\lambda} \geq (<) \frac{1-\rho}{\delta} \frac{(1-\delta^2)(2+\delta)\rho^2}{(2\rho^2 - \delta(1-(2-\rho)\rho))}$ effort is tent-shaped (decreases).

PROOF: See Appendix. ■

As shown in the above corollary, in contrast to when there is a single principle, the sensitivity to own performance can be lowest when output correlation is low.

4.2. Multi-Agent Firms

Now suppose each principal manages a group or team of agents who are benchmarked to a broader population. These teams might correspond to workers in similar occupations in separate firms (e.g. textile workers at nearby plants, or executives at competing firms within an industry), or even workers in separate departments within a firm (if their teams are managed independently).

Specifically, we let $n+1$ be the size of the total population of peers as before, and assume each principal manages a team of $\hat{n}+1$ agents, with $\hat{n} \in [0, n]$. Note that this setting generalizes the cases we have analyzed this far: When $\hat{n} = n$ we are in the single principal setting of Section 3, whereas the single agent setting of Section 4.1 corresponds to $\hat{n} = 0$.

We assume that each principal proposes contracts to the members of his team independently. Agents know the contracts of other members of their team, but don't observe (and so must anticipate in equilibrium) the contracts used at different firms. We assume for simplicity that the strength of peer effects is the same both within and across teams (though it would be straightforward to allow for peer effects to be stronger within teams).

When principals set the contracts for their team, the same distortion arises as in **PROPOSITION III** – each principal ignores the cost of paying higher wages on the utility of outsiders, and moreover perceives a benefit from changing effort in a way that might reduce the expected wage of outsiders. The distortion is mitigated, however, as the fraction of workers who are outsiders diminishes as team size increases.

PROPOSITION IV. Suppose each principal contracts with $\hat{n}+1$ agents. Contracts are public within the team but private across teams. Then the equilibrium symmetric contract has

$$(\alpha^{\hat{n}}, \beta^{\hat{n}}, x^{\hat{n}}, y^{\hat{n}}, a^{\hat{n}}) = \frac{1}{1 - \hat{\delta}(1 - y^s)} (\alpha^s, \beta^s, x^s, y^s, a^s). \quad (28)$$

where

$$\hat{\delta} = \frac{n - \hat{n}}{n - \hat{n}\delta} \delta \leq \delta.$$

Hence equilibrium effort is increasing with δ and distorted upward as in **PROPOSITION III**, but to an extent which is decreasing in \hat{n} (and disappears when $\hat{n} = n$).

PROOF: Let w_t be the wage paid to a member of the principal's team, and w_{-t} the average wage of a non-member. Then, because the fraction \hat{n}/n of the agent's peers are on the same team, the participation constraint (23) becomes

$$\begin{aligned}
E[w_t] &= \frac{\hat{n}}{n} \delta E[w_t] + (1 - \frac{\hat{n}}{n}) \delta E[w_{-t}] + (1 - \delta) [c_0 + \psi(a_i) + \lambda \text{Var}(c_i)] \\
E[w_t] &= \frac{n\delta - \hat{n}\delta}{n - \hat{n}\delta} E[w_{-t}] + \frac{n - n\delta}{n - \hat{n}\delta} [c_0 + \psi(a_i) + \lambda \text{Var}(c_i)] \\
E[w_t] &= \hat{\delta} E[w_{-t}] + (1 - \hat{\delta}) [c_0 + \psi(a_i) + \lambda \text{Var}(c_i)]
\end{aligned}$$

Equation (28) then follows exactly as in the proof of **PROPOSITION III**. Because

$$\hat{\delta}(1 - y^s) = \frac{n - \hat{n}}{n - \hat{n}\delta} \times \delta(1 - y^s),$$

and both terms are increasing with δ (the latter from (27)), effort increases with δ if $\hat{n} < n$.

Because $\hat{\delta}$ is decreasing in \hat{n} , the distortion declines with team size. ■

Again, note that (28) nests both of our earlier results. The single principal setting corresponds to $\hat{n} = n$, while many principal setting in **PROPOSITION III** corresponds to $\hat{n} = 0$.

5. Commitment, Disclosure and Renegotiation

Thus far we have assumed that each principal discloses the incentive contracts used within the firm, and cannot privately alter individual contracts. But suppose individual agents can attempt to renegotiate with the principal, and the principal cannot commit to refrain from such renegotiation. If contract alterations are possible, and can be hidden from other agents within the firm, then in equilibrium we should require that contracts be renegotiation-proof.

If the principal and agent renegotiate privately, they will ignore the impact of their wage choice on the utility of other agents, as well as try to lower the wage of others through the performance benchmark, just as in single agent setting of Section 4.1. Moreover, there is now an added benefit to the principal: lowering the wage of other agents within the same firm contributes directly to the principal's profits.

But while there is an incentive to renegotiate, the opportunity to do so must hurt the principal ex-ante. In equilibrium, other agents within the firm will anticipate the renegotiated contract and seek commensurate terms. In other words, because the renegotiation-proof contract could always be proposed in an environment with disclosure, allowing hidden renegotiation only constrains the principal.

But while each principal is individually worse off with hidden contracting, the equilibrium consequence of renegotiation is less clear: constraining contracts in this way might reduce some of the inefficiency that arose from independent contracting in Section 4. The following result characterizes the equilibrium outcome when hidden renegotiation cannot be prevented:

PROPOSITION V. Suppose each principal privately contracts with a team of $\hat{n} + 1$ agents. Then the equilibrium symmetric and renegotiation-proof contract has

$$(\alpha^{rp}, \beta^{rp}, x^{rp}, y^{rp}, a^{rp}) = \frac{1}{1 - \delta(1 - y^s) + \hat{n}y^s} (\alpha^s, \beta^s, x^s, y^s, a^s). \quad (29)$$

Equilibrium effort is above the single principal case when $\delta > 0$, and increases with δ . Effort increases in \hat{n} if $\delta < \theta_{n,\rho}$ and decreases in \hat{n} if $\delta > \theta_{n,\rho}$. Welfare varies inversely with effort. Finally, in the special case $\hat{n} = n$, so that there is a single principal who contracts privately with each agent,

$$\frac{1}{1 - \delta(1 - y^s) + ny^s} = \frac{1}{1 - ((1 - \alpha^s)\delta + \alpha^s\theta_{n,\rho})}.$$

PROOF: Each principal has a potential incentive to contract privately with one agent so as to reduce the wage paid to the \hat{n} other agents under his span of control. Thus, the principal's problem when considering such a deviation changes from (24) to include this benefit:

$$\max_{(x_i, y_i, a_i)} a_i - ((\hat{n} + \delta)E[w_{-i}] + (1 - \delta)[\psi(a_i) + \lambda \text{Var}(c_i)]) \quad s.t. \text{ (IC)} \quad (30)$$

Following the same solution method as in **PROPOSITION III**, at the optimum we have

$$y^{rp} = \left(\frac{1 - (\hat{n} + \delta)y^{rp}}{1 - \delta} \right) y^s.$$

We can solve for y^{rp} as

$$y^{rp} = \frac{1}{1 - \delta(1 - y^s) + \hat{n}y^s} y^s,$$

and again this same scaling factor will apply to each of the contract variables. Next, because $\alpha^s < 1$ and $\theta_{n,\rho} \in [0, 1]$, we have

$$\frac{\delta}{\hat{n} + \delta} \geq \frac{\delta}{n + \delta} > y^s = -\alpha^s \left(\frac{\theta_{n,p} - \delta}{n + \delta} \right) > -\frac{1 - \delta}{n + \delta} \geq -\frac{1 - \delta}{\hat{n} + \delta}, \quad (31)$$

so that the scaling factor exceeds 1 for all δ . For the comparative statics with respect to δ , note that

$$\begin{aligned} \delta(1 - y^s) - \hat{n}y^s &= \delta - (n + \delta)y^s + (n - \hat{n})y^s \\ &= \delta - (n + \delta) \left(-\alpha^s \left(\frac{\theta_{n,p} - \delta}{n + \delta} \right) \right) + (n - \hat{n})y^s \\ &= \delta + \alpha^s (\theta_{n,p} - \delta) + (n - \hat{n})y^s \\ &= (1 - \alpha^s)\delta + \alpha^s \theta_{n,p} + (n - \hat{n})y^s \end{aligned} \quad (32)$$

is strictly increasing in δ . The comparative statics with respect to \hat{n} follow since $y^s < (>) 0$ iff $\delta < (>) \theta_{n,p}$. Finally, the special case of $\hat{n} = n$ is implied by (32). ■

Comparing (29) with (28), we note two effects. First, $\hat{\delta}$ is replaced with δ because the principal does not consider the impact of the renegotiation with one agent on the utility of other agents on the team when the renegotiation is hidden. Second, the new term $\hat{n}y^s$ captures the principal's gain from manipulating the performance benchmark to lower wages for the rest of the team. In particular, note that the renegotiation-proofness constraint creates a distortion when $\hat{n} > 0$ even if $\delta = 0$. That is because even without relative wealth concerns, the principal can manipulate the wages of other agents on his team by manipulating the RPE benchmark.

On the other hand, when relative wealth concerns are strong and $\delta > \theta_{n,p}$, then $y^s > 0$ and each agent's wages are positively related to the output of others. In that case, renegotiation-proofness implies that productivity will decrease with team size, but efficiency will improve. As a result, with multi-agent firms, lack of commitment and hidden contracting with firms can improve efficiency when relative wealth concerns are strong:

PROPOSITION VI. Suppose each principal manages a team of $\hat{n} + 1$ agents, with $0 < \hat{n} < n$. Then for δ close to zero, public contracting within the team dominates private contracting. However, when δ is close to one, private contracting is more efficient.

PROOF: Because in all cases effort weakly exceeds the optimum from **PROPOSITION I**, the more efficient outcome will be the one that leads to the lowest effort level. To compare effort levels, we need only to compare the scale factors in (28) and (29). Thus, public contracting dominates private contracting if and only if

$$1 - \hat{\delta}(1 - y^s) > 1 - \delta(1 - y^s) + \hat{n}y^s.$$

We can rewrite this as

$$\hat{n}y^s < \frac{\hat{n} - \hat{n}\delta}{n - \hat{n}\delta} \delta(1 - y^s),$$

or more simply

$$y^s < \frac{(1 - \delta)\delta}{n - \hat{n}\delta} (1 - y^s). \quad (33)$$

Recall that $y^s = -\alpha^s \left(\frac{\theta_{n,p} - \delta}{n + \delta} \right)$ and $\theta_{n,p} \in (0, 1)$. Hence $y^s < 0$ for $\delta = 0$ and $y^s > 0$ for $\delta = 1$.

The result then follows since the right-hand side of (33) converges to zero for $\delta = 0$ or 1. ■

Another natural comparison is the case of a single principal who cannot commit to the case of independent principals each managing a single agent. The following result is immediate:

COROLLARY V.A If a single principal can privately renegotiate, wages and effort are higher, and profits are lower, than with independent principals if $\delta < \theta_{n,p}$ so that $y^s < 0$.

The converse holds when $\delta > \theta_{n,p}$ and therefore $y^s > 0$.

6. External Disclosure

Until now we have considered only the possibility of disclosure of contracts within a team (i.e. a single principal's span of control). In this section we consider the case in which contracts are disclosed externally, so that all agents are aware of the contracts held by all others.

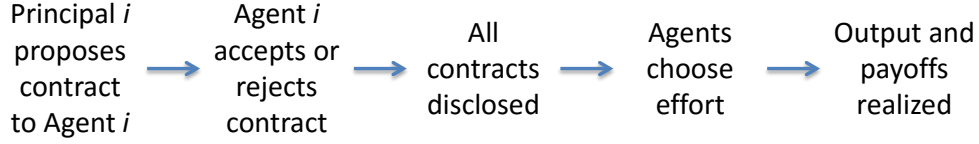


Figure 7: Many Independent Principals with Externally Disclosed Contracts

Consider the case in which there is an independent principal setting the contract for each agent (i.e. teams are size one, and $\hat{n} = 0$). Then taking (m_{-i}, x_{-i}, y_{-i}) as given, the optimal contract for principal and agent i solves

$$\max_{(x_i, y_i, a_i, a_{-i})} a_i - \left(\delta E[w_{-i}] + (1 - \delta) [\psi(a_i) + \lambda \text{Var}(c_i)] \right) \quad s.t. \quad (IC) \quad (34)$$

The difference between this case and (24) of Section 4 is that principal and agent i recognize that once their contract is disclosed, other agents will adjust their actions a_{-i} accordingly. In other words, they will solve for their optimal contract taking into account the (IC) constraint for all agents, not solely that for agent i .

Note that the (IC) constraint for other agents can be written as

$$a_{-i} = k \left(\frac{x_{-i} - \delta \left(\frac{n-1}{n} y_{-i} + \frac{1}{n} y_i \right)}{1 - \delta} \right). \quad (35)$$

Therefore, y_i will affect the actions of other agents, and principal i will have an incentive to manipulate a_{-i} through this channel. Raising y_i will induce other agents to reduce their effort in order to reduce the benefit to agent i . But when other agents reduce their effort, they will also receive a lower wage (as long as $x_{-i} + (n-1)y_{-i} > 0$), and this lowers the cost of compensating agent i . As a result, equilibrium contracts will no longer choose y_i , or equivalently β_i , to minimize variance as in **LEMMA 3**, but will instead involve higher y and lower overall effort. Moreover, the solution will no longer be a simple rescaling of the solution in **PROPOSITION I**, as we can see in the following result.

PROPOSITION VII. Suppose independent principals each manage a single agent, and all contracts are disclosed prior to agents choosing effort. Then the equilibrium contract is symmetric and satisfies:

$$\frac{y^{md}}{x^{md}} = \frac{y^s + B}{x^s - (n-1)B} \quad (36)$$

and

$$\alpha^{md} = \alpha^m - \frac{\delta^2 (1-\delta\rho) \left(\frac{\delta}{2\lambda k^{-1}} + \rho(n+\delta)(1-\delta) \right)}{(1-\delta)W_1(W_1+W_2)}, \quad (37)$$

where

$$B = \left(\frac{\delta\alpha^s}{n} \right)^2 \frac{\theta_{n,\rho} \sigma_{n,\rho}^2}{(n+\delta)(1-\alpha^s)\rho} \geq 0$$

$$W_1 = \left[n + \left(\frac{1}{\alpha^s} - 1 \right) (n+\delta) \right] (1-\delta)(1-\rho+n\rho) + \delta(1-\rho) > 0.$$

$$W_2 = \left(\frac{\delta^2}{n} \right) \left(1-\delta-n(1-\rho) - \frac{n-1}{2\lambda k^{-1}} \right)$$

PROOF: We can calculate

$$\frac{\partial}{\partial a_i} (a_i - \delta E[w_{-i}] - (1-\delta)\psi(a_i)) = 1 - \delta y_j - (x_i - \delta y_j) = 1 - x_i \quad (38)$$

Then, using the (IC) constraint for a_i and the expression for $Var(c_i)$, we have the following first order conditions (which are sufficient given the strict concavity of the objective function) for the optimal choice of (x_i, y_i) given (x_j, y_j) :

$$(1-x_i) - 2\lambda k^{-1} \left[(1-\rho)(x_i - \delta y_j) + \rho(x_i - \delta x_j + n y_i - \delta n y_j) \right] = 0 \quad (39)$$

$$\left(\frac{\delta}{n} \right)^2 (x_j + (n-1)y_j) - 2\lambda k^{-1} \left[\frac{1}{n} (1-\rho)(n y_i - \delta x_j - (n-1)\delta y_j) + \rho(x_i - \delta x_j + n y_i - \delta n y_j) \right] = 0 \quad (40)$$

The first order conditions above are identical to the first order conditions for the optimal contract in **PROPOSITION III** with the exception of the first term in (40). This term arises because of the effect of y_i on the effort choice a_j of other agents, which impacts their expected wage:

$$E[w_{-i}] = E[w_j] = m_j + x_j a_j + y_j (a_i + (n-1)a_j)$$

It is this term that implies that β_i (which is determined by y_i) will not be chosen as the optimal hedge to minimize the variance of c_i , but instead will be distorted to impact others' effort.

Solving (39) and (40) for (x_i, y_i) , we find the following "reaction functions":

$$x_i = \alpha_s + \delta y_j (1 - \alpha_s) - (x_j + (n-1)y_j) \frac{\delta^2}{n^2} \alpha_s \theta_{n,\rho} \quad (41)$$

$$n y_i = (x_j + (n-1)y_j) \left(\delta + \frac{\delta^2}{n^2} \left(\frac{1 - \alpha_s \theta_{n,\rho} \rho}{1 - \alpha_s} \right) \frac{\alpha_s \theta_{n,\rho}}{\rho} \right) - (1 - \delta y_j) \alpha_s \theta_{n,\rho} \quad (42)$$

These reaction functions are equivalent to the reaction functions in the setting of **PROPOSITION III** upon replacing the δ^2 terms with zeros. Finally, we solve for a symmetric equilibrium by solving (41) and (42) with $(x_i, y_i) = (x_j, y_j)$. After much tedious algebra one can solve for

$$x^{md} = x^m + \frac{\delta^2 (1 - \delta \rho) (1 + (n-1)\rho) (\delta - \theta_{n,\rho})}{W_1 (W_1 + W_2)} \quad (43)$$

$$y^{md} = y^m + \frac{\delta^2 (1 - \delta \rho) \left(1 - \delta \rho + \frac{1}{2\lambda k^{-1}} \right)}{W_1 (W_1 + W_2)} \quad (44)$$

In the appendix we show that the equilibrium must be symmetric. ■

Recent regulation has increased disclosure requirements of CEO compensation.¹³ Comparing the setting with publically disclosed contracts to the one with undisclosed contracts shows that publicly disclosed contract generally imply higher relative sensitivity compared to when contracts are undisclosed (see (36)). Furthermore, as the KUI incentives increase effort at first increases, yet to a lesser extent than with undisclosed contracts, but then decreases. With

¹³ In December 2006, the Securities and Exchange Commission (SEC) issued new enhanced disclosure requirements on CEO compensation (for details see <http://www.sec.gov/rules/final/2006/33-8732a.pdf>).

sufficiently high KIJ incentives the equilibrium contract distortions induce an effort level that is less than the one with a single principal. The principal's payoff is also non-monotonic, initially decreasing as effort rises, then increasing as effort becomes closer to second-best, and finally decreasing as effort drops further. Ultimately, if KIJ effects are sufficiently strong, effort collapses as shown in Figure 8. Indeed, there exists a cutoff such that if and only if δ exceeds the cutoff, then equilibrium profits are lower when contracts are externally disclosed. Thus, external disclosure requirements are unique in creating the possibility that relative wealth concerns may lead to equilibrium effort and productivity below that of the standard contracting environment.

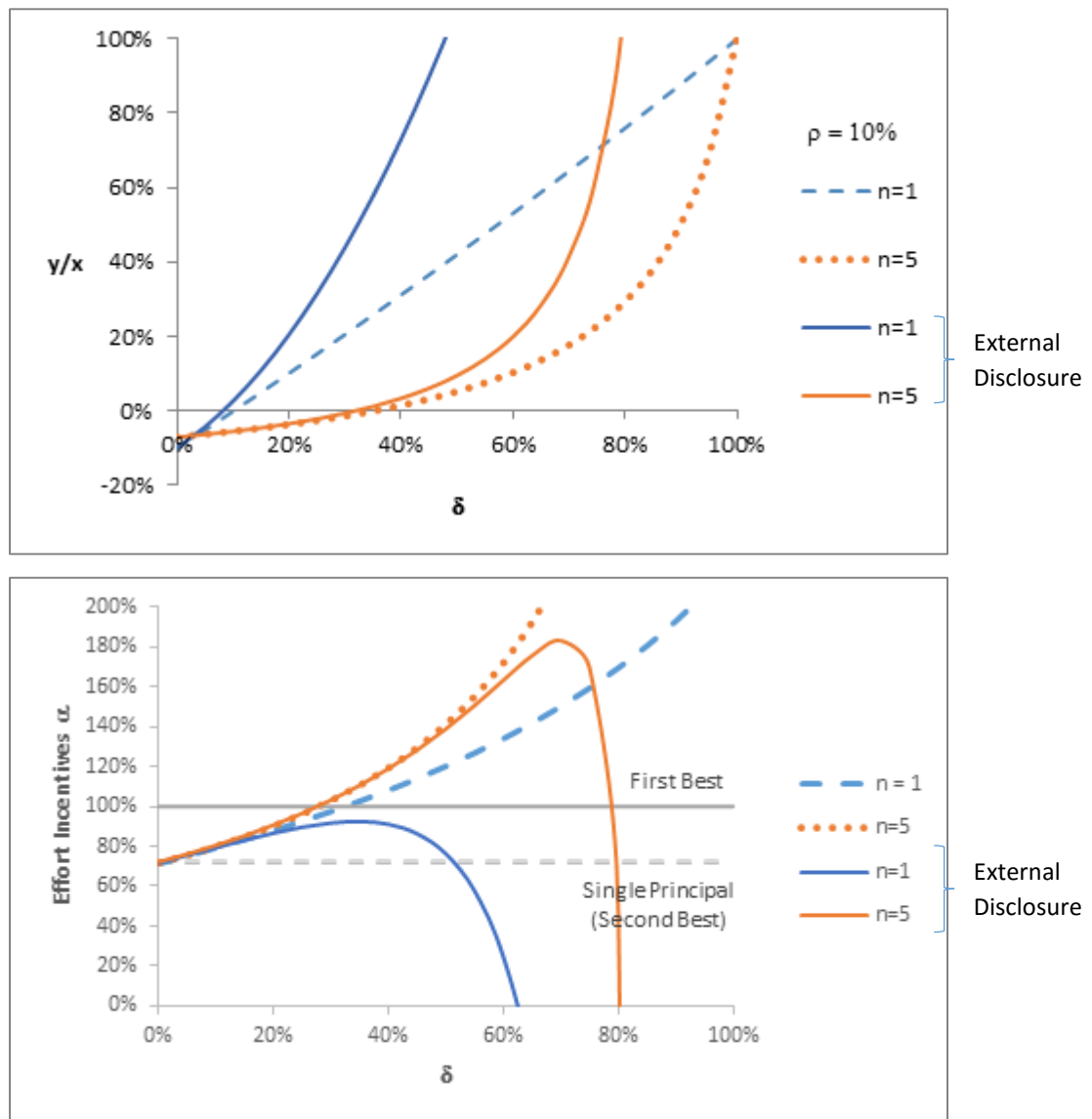


Figure 8: External Disclosure Leads to Higher Pay for Luck and Lower Effort

7. Conclusion

In this paper we have extended a standard moral hazard optimal contracting framework to a setting in which agents care about both their absolute wage, as well as how their wage compares to that of their peers. We show that as the strength of this Keeping Up with the Joneses (KUJ) component of preferences increases in importance, optimal contracts deviate from relative performance evaluation and thus exhibit “pay for luck.” In the extreme, agents are paid only on the basis of aggregate output, rather than for their individual performance.

Surprisingly, despite the fact that contracts appear to provide suboptimal incentives, we show that when there is a single principal, efficiency is unaffected by KUJ preferences. Rather, optimal contracts hedge the added risk from relative wage concerns, and effort, average wages, and profitability are unaffected. The correlation between wages rises, however, with the degree of relative wage concerns.

When there are multiple principals, contracts display the same relative sensitivity to aggregate versus individual output as with a single principal. But now, contracts fail to account for the externality that an increase in output has on the welfare of other agents. As a result, principals use inefficiently high-powered incentives, and agents work too hard. In equilibrium, they demand higher average wages to compensate for this effort, reducing firm profits.

Finally, we consider settings when principals manage teams of agents, and when there are different disclosure rules regarding contracts. We show that when KUJ effects are weak, private renegotiation increases distortions, but when KUJ effects are strong, equilibrium efficiency is enhanced if principals negotiate privately with individual agents. Finally, when contracts are disclosed externally to agents on other teams, effort is reduced and incentives may collapse.

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9. Appendix

LEMMA A.1: With multiple principal-agent pairs and private contracting the equilibrium is unique.

PROOF:

$$w_{-i} = m_{-i} + \frac{1}{n} \sum_{j \neq i} x_j q_j + y_{-i} q_i + \frac{n-1}{n} \sum_{j \neq i} y_{-(i,j)} q_j \quad (45)$$

where

$$y_{-(i,j)} \equiv \frac{1}{n-1} \sum_{k \neq i,j} y_k \quad (46)$$

Implying that

$$w_i - \delta w_{-i} = m_i - \delta m_{-i} + (x_i - \delta y_{-i}) q_i + \frac{1}{n} \sum_{j \neq i} (n y_i - \delta (x_j + (n-1) y_{-(i,j)})) q_j \quad (47)$$

$$\text{Var}[c_i] = \frac{1}{(1-\delta)^2} \text{Var}[w_i - \delta w_{-i}]$$

where

$$\begin{aligned} \text{Var}[w_i - \delta w_{-i}] &= (x_i - \delta y_{-i})^2 + 2(x_i - \delta y_{-i}) (y_i - \delta (x_{-i} + (n-1) y_{-(i,j)})) \rho \\ &+ \frac{1}{n^2} \sum_{j \neq i} \sum_{k \neq i,j} (y_i - \delta (x_j + (n-1) y_{-(i,j)})) (y_i - \delta (x_k + (n-1) y_{-(i,k)})) \rho \\ &+ \frac{1}{n^2} \sum_{j \neq i} (y_i - \delta (x_j + (n-1) y_{-(i,j)}))^2 \end{aligned} \quad (48)$$

Using the above expressions, the optimal solution of optimization (22) is obtained by imposing IC_i and taking first order conditions for the optimal choice of (x_i, y_i) given $\{(x_j, y_j)\}_{j \neq i}$.

After rearranging the first order conditions we find the following “reaction functions”

$$\begin{cases} x_i = \alpha^s + \frac{\delta}{n} (1 - \alpha^s) n y_{-i} \\ n y_i = \alpha^s \theta_{n,\rho} + \delta x_{-i} + \frac{\delta}{n} n y_{-i} ((n-1) - \alpha^s \theta_{n,\rho}) \end{cases} \quad (49)$$

Taking the difference between the reaction functions of principals i and k , and rearranging each of the two differences yields

$$\begin{cases} x_k - x_i = \frac{-\delta}{n}(1 - \alpha^s)(y_k - y_i) \\ -n(y_k - y_i) = \frac{\delta}{n}(x_k - x_i) + \frac{\delta}{n}((n-1) - \alpha^s \theta_{n,\rho})(y_k - y_i) \end{cases} \quad (50)$$

Plugging the first equation into the second and rearranging yields

$$0 = \left[\frac{-\delta^2}{n^2}(1 - \alpha^s) + \frac{\delta}{n}((n-1) - \alpha^s \theta_{n,\rho}) + n \right] (y_k - y_i) \quad (51)$$

Note that for $n > 1$ clearly $[] > 0$.

For $n=1$:

$$[] = -\delta^2(1 - \alpha^s) - \delta \alpha^s \theta_{n,\rho} + 1 = -\delta^2(1 - \alpha^s + \alpha^s \theta_{n,\rho}) + 1 > -\delta^2 + 1 > 0$$

Finally, $[] > 0$ implies that $y_k = y_i$ which from the first equation implies $x_k = x_i$ as well.

■

LEMMA A.2: Suppose independent principals each manage a single agent, and all contracts are disclosed prior to agents choosing effort. Then the equilibrium must be symmetric.

PROOF: Allowing contract to potentially differ across principals, and using the expressions in Equations (43)-(46) the “reaction functions” corresponding to Equations (39) and (40) take the form

$$x_i = \alpha_s + \delta y_{-i}(1 - \alpha_s) - (x_{-i} + (n-1)y_{-i}) \frac{\delta^2}{n^2} \alpha_s \theta_{n,\rho} \quad (52)$$

$$n y_i = (x_{-i} + (n-1)y_{-i}) \left(\delta + \frac{\delta^2}{n^2} \left(\frac{1 - \alpha_s \theta_{n,\rho} \rho}{1 - \alpha_s} \right) \frac{\alpha_s \theta_{n,\rho}}{\rho} \right) - (1 - \delta y_{-i}) \alpha_s \theta_{n,\rho} \quad (53)$$

Taking the difference between the reaction functions of principals i and k , and rearranging each of the two differences yields

$$(x_k - x_i) \left(\frac{1}{n} \frac{\delta^2}{n^2} \alpha_s \theta_{n,\rho} - 1 \right) = (y_k - y_i) \left(\frac{\delta}{n} (1 - \alpha_s) - \frac{n-1}{n} \frac{\delta^2}{n^2} \alpha_s \theta_{n,\rho} \right)$$

$$-n(y_k - y_i) = ((x_k - x_i) + (n-1)(y_k - y_i)) \frac{1}{n} \left(\delta + \frac{\delta^2}{n^2} \left(\frac{1 - \alpha_s \theta_{n,\rho} \rho}{1 - \alpha_s} \right) \frac{\alpha_s \theta_{n,\rho}}{\rho} \right) + \frac{\delta}{n} (y_k - y_i) \alpha_s \theta_{n,\rho}$$

Using the first equation to plug into the second and then simplifying yields

$$0 = \left[\frac{\left(\frac{\delta}{n} (1 - \alpha_s) - (n-1) \right)}{\left(\frac{1}{n} \frac{\delta^2}{n^2} \alpha_s \theta_{n,\rho} - 1 \right)} \frac{1}{n} \left(\delta + \frac{\delta^2}{n^2} \left(\frac{1 - \alpha_s \theta_{n,\rho} \rho}{1 - \alpha_s} \right) \frac{\alpha_s \theta_{n,\rho}}{\rho} \right) + \frac{\delta}{n} \alpha_s \theta_{n,\rho} + n \right] (y_k - y_i)$$

Note that for $n > 1$ clearly $[] > 0$.

For $n = 1$, $\theta_{n,\rho} = \rho$ and

$$\begin{aligned} [] &= \frac{-\delta(1 - \alpha_s)}{(1 - \delta^2 \alpha_s \theta_{n,\rho})} \left(\delta + \delta^2 \left(\frac{1 - \alpha_s \theta_{n,\rho} \rho}{1 - \alpha_s} \right) \frac{\alpha_s \theta_{n,\rho}}{\rho} \right) + \delta \alpha_s \theta_{n,\rho} + 1 \\ &= \frac{-\delta^2(1 - \alpha_s) - \delta^3(1 - \alpha_s \rho^2) \alpha_s + (1 - \delta^2 \alpha_s \rho)(\delta \alpha_s \rho + 1)}{(1 - \delta^2 \alpha_s \rho)} \\ &= \frac{(1 - \delta)(1 + \delta(1 + \alpha_s(\delta + \rho)))}{(1 - \delta^2 \alpha_s \rho)} > 0 \end{aligned}$$

Finally, $[] > 0$ implies that $y_k = y_i$ which from the first equation implies also that $x_k = x_i$

PROOF OF COROLLARY 1C: From (18), α is decreasing in λ , and from (19) this effect is the only impact on x and y , implying the first result. Next, an increase in ρ decreases $\sigma_{n,\rho}^2$, which raises α and effort. An increase in ρ also increases $\theta_{n,\rho}$, and so y/x declines because

$$y/x = \frac{-\left(\frac{\theta_{n,\rho} - \delta}{n + \delta} \right)}{1 - \delta \left(\frac{n + \theta_{n,\rho}}{n + \delta} \right)} = \frac{-\left(\frac{\theta_{n,\rho} - \delta}{n + \delta} \right)}{(1 - \delta) - \delta \left(\frac{\theta_{n,\rho} - \delta}{n + \delta} \right)} = \frac{1}{\delta - \frac{(1 - \delta)(n + \delta)}{\theta_{n,\rho} - \delta}}.$$

Taking a derivative of y with respect to ρ , while letting $h = 2\lambda k^{-1}$ and simplifying yields

$$-\frac{n}{n+\delta} \frac{(1+h-2\delta hx+h(\delta+n(1-\delta))x^2)}{(-1+x-nx+h(-1+x)(1+nx))^2},$$

which has the same sign as

$$-\left(\frac{1}{h}+1-2\delta x+(\delta+n(1-\delta))x^2\right) < -(1-2x+x^2) < 0.$$

The sign of the derivative of x with respect to ρ is the same as

$$-\delta(k+2\lambda)+4(n(1-\delta)+\delta)\lambda\rho-2(\delta+n(1-\delta)-n^2(1-\delta))\lambda\rho^2 \quad (54)$$

Note that at $\rho=0$ this expression is negative, and at $\rho=1$ it reduces to

$$2n(1+n)\lambda-(2n(1+n)\lambda+k)\delta \quad (55)$$

Expression (55) is decreasing in δ and equals zero at $\delta = \frac{n^2+n}{n^2+n+k/(2\lambda)}$. So if

$\delta < \frac{n^2+n}{n^2+n+k/(2\lambda)}$, then the relation between x and ρ is u-shaped. Otherwise, to be sure it is decreasing, we must make sure the maximum of (54) for $\rho \in (0,1)$ is negative.

If the coefficient on ρ^2 in (54) is non-negative, the result is clear. If the coefficient on ρ^2 in (54) is negative, the maximum of (54) is obtained at $\rho = \frac{\delta+n(1-\delta)}{\delta+n(1-\delta)-n^2(1-\delta)}$, which is larger than 1.

The sign of the derivative of ny/x with respect to n is the same as the sign of

$$-n^2(1-\delta)\rho-2n(1-\delta)\delta\rho+\delta^2(1-\rho)$$

For $\rho=0$ this expression is positive, unless $\delta=0$. For $\rho>0$, it is evident that for n large enough this expression is negative, and that when equating this expression to zero and solving for n at least one of the solutions is negative. Comparing the value of ny/x for $n=2$ to the

value for $n=1$ yields after some algebra that it is larger at $n=2$ iff $\rho < \frac{\delta^2}{\delta+2(1-\delta^2)}$.

The change in effort as n increases follows since effort is second-best, and if $\rho>0$, then aggregate peer output becomes a more informative signal.

■

PROOF OF COROLLARY III.B: Apart for sensitivity of x with respect to ρ and sensitivity of effort with respect to n , results follow immediately from taking derivatives of expressions in **PROPOSITION III.**

The sign of the derivative of x with respect to ρ is the same as the sign of

$$((n-1)n(1-\delta)-\delta)\rho^2 + 2(n(1-\delta)+\delta)\rho - \delta$$

Since this expression is quadratic in ρ , is negative at $\rho=0$ and positive at $\rho=1$, the result follows.

The sign of the derivative of effort with respect to n is the same as the sign of

$$-n^2(1-\delta)(k\delta - 2\lambda(1-\rho))\rho^2 + 2n\delta(1-\rho)\rho(k\delta + 2(1-\delta)\lambda\rho) + \delta^2(1-\rho)(k + 2(1-\delta)\lambda\rho^2)$$

This expression is quadratic in n , is zero at $\rho = \delta = 0$, positive when $\rho = 0$ and $\delta > 0$ or $\frac{k}{2\lambda} \leq \frac{1-\rho}{\delta}$. When $\frac{k}{2\lambda} > \frac{1-\rho}{\delta}$ the n^2 term is negative, and directly comparing effort levels at $n=1$ and $n=2$ yields the condition as to when the sensitivity with respect to n is tent shaped or decreasing.

■