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So What Orders Do Informed Traders Use?*

We analyze informed traders' equilibrium choice of limit or market orders. We show that even after incorporating an order's price impact, not only may informed traders prefer to use limit orders, but also the probability of submitting limit orders can be so high that in equilibrium limit orders convey more information than market orders. We further show that the horizon of the private information is critical for this choice and is positively related to the probability of using limit orders. Our empirical analysis suggests that informed traders do prefer to use limit orders and that limit orders are indeed more informative.

I. Introduction

Informed trading significantly affects market price and transaction dynamics and has thus become one of the most important issues considered in the microstructure literature. In most financial markets (e.g., New York Stock Exchange [NYSE], NASDAQ, Paris Bourse, Tokyo, and Toronto), for any order a trader decides to submit, she can further choose to submit it as a limit order or as a market order. However, despite the importance of informed trading and the almost universal prevalence of order type choice, the decision on the optimal order type by an *informed* trader has thus far been explored only in a partial equilibrium setting.

This article presents a simple, modified Glosten-Milgrom (1985) type equilibrium model to investigate the decision of informed traders on whether to use limit or market orders. Specifically, the market for an asset consists of risk-neutral agents: informed traders, uninformed traders, and a market maker. Before the initial trading the informed traders learn the true asset

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value. This information will be revealed to the public at a random future time, implying a random horizon for the information. Any randomly chosen trader (informed or uninformed) can choose to submit a limit order or a market order, and the market maker posts quotes that yield zero profit in conditional expectation.

In contrast to the almost standard assumption in the theoretical literature that informed traders use only market orders, we demonstrate that even after accounting for the equilibrium price impact of an order, if the probability that the information is long lived is high, then informed traders are more likely to place limit orders than market orders. In addition, we show that under some reasonable conditions not only do informed traders prefer limit orders, but the probability that limit orders come from informed traders can be so high that limit orders actually convey more information than market orders.

Our analysis highlights the fact that the expected horizon of the informed traders' private information is critical for the choice of limit orders versus market orders. We find that the information horizon is positively related to the use of limit orders. Intuitively, a market order is guaranteed execution, but if the order size is larger than the prevailing quote depth, then the trader bears price risk. However, a limit order has no price risk and in general implies a price improvement relative to a market order, but it does have execution risk. As the expected horizon of private information increases, the probability that a limit order would be hit becomes greater, thereby decreasing the disadvantage of having uncertain execution. As a result, the longer the information horizon, the more attractive limit orders become to informed traders.

In addition, we also show that the information horizon is negatively related to the bid-ask spread. This is mainly driven by the fact that, as the horizon increases, informed traders submit market orders less often; thus the probability that a submission of a market order is information-based decreases. Accordingly, the market maker faces less of an adverse selection problem.

Given that limit orders may convey more or less information than market orders depending on fundamental parameters, it becomes an empirical question which order type conveys more information on average in an actual market. Using the TORQ database, we show that limit orders convey more information than market orders about future prices. This implies that informed traders prefer to submit limit orders on average. Moreover, we also show that specialists on the NYSE indeed correctly perceive this informativeness of limit orders.

Using experimental asset markets, Bloomfield, O'Hara, and Saar (2003) investigate the order choice of informed traders. Interestingly, they also find that informed traders use more limit orders than liquidity traders do. Furthermore, they show that liquidity provision shifts over time, with informed traders increasingly providing liquidity in the markets. Our theoretical and empirical analysis complements their experimental study.

In a partial equilibrium setting, Angel (1994) develops a single-period model to investigate the choice of an informed trader who is forced to purchase a security. He assumes exogenously fixed bid and ask prices and an exogenously fixed order flow process in which buy orders and sell orders arrive with equal probability. He concludes that most informed traders would prefer market orders since they believe that the stock is going up, the probability of a limit order execution is low, and the loss from nonexecution is high. Furthermore, he argues that informed traders are less likely than liquidity traders to use limit orders. However, for a sufficiently high exogenously specified order flow process, he shows that increasing the spread by decreasing the bid while holding the ask price fixed makes market orders relatively less attractive and can lead the investor to prefer placing a limit order in some cases. However, it is not clear whether this conclusion would hold in equilibrium since both quotes and order flows would be endogenous.

Harris (1998) investigates dynamic order submission strategies of some stylized traders. In his model, liquidity traders need to purchase a fixed number of shares by a given deadline. Informed traders can purchase a fixed number of shares by the deadline but can also choose not to trade. He shows that traders who face early deadlines and traders who have material information that will soon become public are impatient and would typically use market orders. Otherwise, when the deadline is distant and the bid-ask spreads are wide, they submit limit orders to minimize their transaction costs.[1] However, in contrast to our model, he assumes that neither order strategy nor information horizon affects quotes.

The fact that in both these papers, with all else held equal, larger spreads make limit orders more attractive is to be expected because the price improvement benefit of a limit order increases as spreads widen. In contrast, our equilibrium analysis demonstrates that longer-lived information, while leading informed traders to submit more limit orders, also leads to smaller spreads. In other words, in our equilibrium model there may be more limit order submission even when spreads shrink. Furthermore, given the partial equilibrium nature of these models, they are not constructed to address the question of the equilibrium relative propensity to use limit versus market orders by informed and uninformed traders.

Rock (1990) and Glosten (1994) explicitly incorporate informed traders into equilibrium models. However, in contrast to our model, they allow informed traders to place only market orders. Rock conjectures quite forcefully that informed traders will in fact use only market orders, whereas uninformed traders will be the ones placing limit orders and possibly also market orders. His argument relies on the assumption of a short information horizon. Specifically, he argues that a market order enables the investor to take a position before the information leaks out; whereas with a limit order the conditions

1. In Harris (1998), informed traders have information that allows them to construct a one-time estimate of value. These traders expect that their estimate of the pricing error will decay at a constant exponential rate. The actual rate of decay in their pricing error may be different from their expected rate of decay, but they are precluded from learning about this discrepancy. For more details on order strategy, see also Easley and O'Hara (1991).

under which the limit order is executed are unlikely to occur, since other informed traders who use market orders will drive the prices against the informed limit user. Glosten provides a similar justification for restricting informed traders to using only market orders. He argues that if there are enough informed market order users or the depreciation rate of private information is large enough, then informed traders will prefer to use market orders. Our empirical finding that limit orders convey more information then suggests that the information horizon of informed traders is, on average, longer (or the depreciation rate of private information is smaller) than what these models implicitly assume.

Consistent with Rock's conjecture, most models in the literature assume that informed traders place only market orders, with Kumar and Seppi (1993) as one exception. In Kumar and Seppi's one-period model, order flow from liquidity traders is generated by an exogenously specified exact factor model, which implies that the informed trader's order strategy is also restricted to the same factor structure in equilibrium. This restriction largely determines the form of the trader's equilibrium order strategy. In contrast, we do not impose such a restrictive factor structure.

Chakravarty and Holden (1995) analyze the behavior of an informed trader in a single-period call-type market. In this market, the market makers precommit to a bid and an ask.[2] They show that in such a market an informed trader may simultaneously submit a market buy (sell) order and a limit sell (buy) order; the orders may cross with each other, and the limit order acts as a safety net for the market order.

Also closely related are Parlour (1998), Foucault (1999), Foucault, Kadan, and Kandel (2002), and Hollifield et al. (2002). These papers consider the limit order versus market order choice problem for *uninformed* traders who have different valuations on the same asset, based on the trade-off between transaction price and execution risk.

Biais, Hillion, and Spatt (1995), Hamao and Hasbrouck (1995), and Harris and Hasbrouck (1996) examine empirical properties of limit orders in various major markets. While these papers cannot serve as a direct test of our model, some of them do find empirical patterns that are consistent with our model. For example, Biais et al. study the interaction between the order book and the order flow in the Paris Bourse market. They find that following large limit orders, shifts in both bid and ask quotes occur in the direction that is consistent with these orders being informative, which supports one of our model's main implications that limit orders may contain information.

Recent papers that investigate empirically the order choice submission and its relation to the information content of orders include Anand, Chakravarty, and Martell (2004) and Beber and Caglio (2004). Beber and Caglio present

2. In their model, the market makers first quote a bid (ask) at which they precommit to buy (sell) any quantity required; then informed and uninformed traders submit their market and limit orders simultaneously, and finally all relevant orders are executed.

evidence suggesting that informed traders strategically use limit orders to hide their information. Anand et al. show that institutions are more likely to use market orders at the beginning of the day and limit orders toward the end of the day. Individuals, however, tend to behave in the opposite manner. They also present evidence suggesting that institutions are informed and individuals are uninformed.[3]

The rest of the paper is organized as follows. Section II contains the derivation of the model and its predictions. Section III is devoted to testing which order type conveys more information and whether a specialist's perception about the informativeness of the two order types is rational. Concluding remarks are given in Section IV.

II. Can Limit Orders Convey More Information?

In this section we develop a simple Glosten-Milgrom (1985) type equilibrium model that allows traders to optimally choose between market orders and limit orders. This model highlights the following main implications. First, the probability that informed traders place limit orders can be higher than the probability that they submit market orders. Second, limit orders can be more informative than market orders in the sense that it is more likely that limit orders are information based. Using the probability of informed trading as a metric for measuring informativeness of orders has been employed in Easley, Kiefer, and O'Hara (1996), Easley et al. (1996), and subsequent papers that use the probability of informed trading measure.[4]

The model time line is depicted in figure 1. The economy consists of a mass of $1 - \mu$ informed traders, μ uninformed traders, and one competitive market maker. All the participants are assumed to be risk neutral. The unknown value of the asset ($v \in [\underline{v}, \bar{v}]$, where $\bar{v}$ [$\underline{v}$] can possibly be ∞ [$-\infty$]) is drawn from a continuous distribution with density function $g(v)$, which is symmetric around its mean (m). Thus $\underline{v} = m - (\bar{v} - m)$.

There are three trading dates. Each date allows for the (potential) arrival of a single unit order. As is common in a Glosten-Milgrom setting, the market maker posts the quote on each date before traders submit orders so that traders condition their orders on the market maker's quote; the specific trader is chosen probabilistically on the basis of the mass measures of the different agents. Furthermore, as is generally observed in markets with limit orders, time pri-

3. Other recent papers that investigate empirically determinants of order choice submission include Bae, Jang, and Park (2003) and Ellul et al. (2003). These papers find that the placement of limit (market) orders is positively (negatively) related to the spread and volatility and limit orders are more likely late in the trading day.

4. Some of the contexts in which such a measure has been used include measuring informativeness of orders across markets, testing whether differences in information-based trading can explain observed differences in spreads for active and infrequently traded stock, understanding the postannouncement drift, analyzing the informational role of transactions volume in options markets, and examining the effect of price informativeness on the sensitivity of investment to stock price.

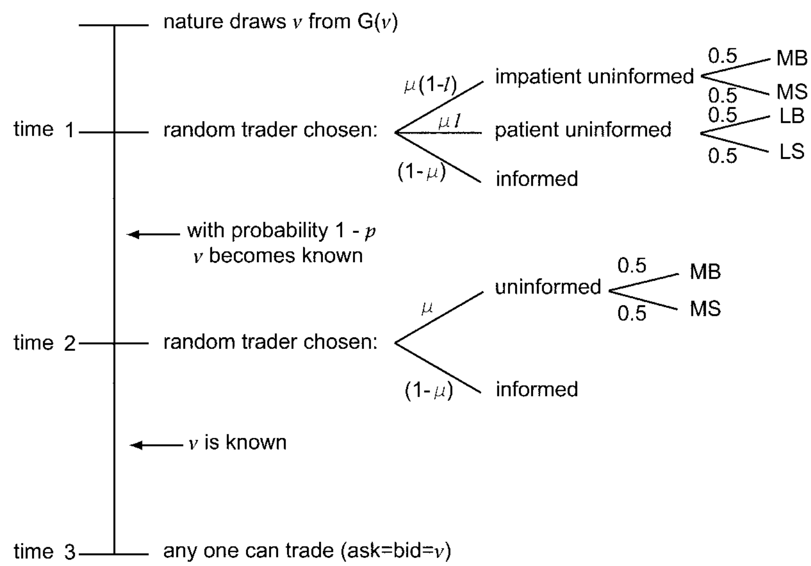

FIG. 1.—Model time line

ority of orders is enforced and market makers are assumed to yield to limit orders. Before the initial trading date, informed traders learn the value v. With probability $1 - p$, the information is short lived and will be revealed to the market by the end of the first trading period. With probability p, the information is long lived and will be revealed to the market only at the end of the second trading period.

On each trading date, given an opportunity to trade, an uninformed liquidity trader needs to buy or sell the asset with probability one-half. This assumption, combined with the symmetry of $g(v)$ around its mean m, allows us to solve for the buy side and obtain the sell side result as an appropriate reflection around m. Although the uninformed traders' motives for purchasing or selling the stock are not directly modeled, we assume that a mass of $1 - l$ of them are impatient so that they use only market orders and a mass of l are patient so that given the opportunity to trade at the initial date they rationally choose an order type to minimize (maximize) the expected purchasing (selling) price, taking into account the market maker's quote-updating process. In particular, in equilibrium we require that the expected buying (selling) price of the asset at the initial date for a patient uninformed trader be strictly below (above) the prevailing ask (bid), so that in equilibrium it is optimal for patient uninformed traders to place limit orders at the initial date. In the remainder of the paper, we refer to this requirement as the participation constraint of the patient uninformed traders. Throughout, unless stated otherwise, we assume $0 < l, \mu, p < 1$.

The ask (bid) price at the initial date is denoted a_1 (b_1) and the ask (bid) at the second trading date is denoted a_2 (b_2). On the third trading date the value of the asset becomes publicly known, which implies $a_3 = b_3 = v$.

Note that, given an opportunity to trade, an informed trader may submit one of the following orders: $O = \{MB, MS, (LB; PB), (LS; PS), NO\}$, where *MB*, *MS*, *LB*, *LS*, and *NO* represent a market buy, market sell, limit buy, limit sell, and no order, respectively, and *PB* and *PS* are limit buy and limit sell prices, respectively. Risk neutrality implies that the informed trader will maximize expected profits.

When an informed trader decides which order to place, in addition to conditioning order choice on the current ask, bid, and value of v, the trader's order submission rule also depends on the market maker's quote-updating process. The quote-updating process is a function of both the order type and the limit price when the order is a limit order, as well as other market characteristics such as the fraction of patient uninformed traders. Specifically, the competitiveness and risk neutrality of the market maker imply that in equilibrium the ask (bid) price must be the expected asset value conditional on a market buy (sell) and on the previous order type and previous order price when it was a limit. Specifically, the market maker's expected profit on each trade is zero. On date 1 he makes a profit on the impatient uninformed traders and loses to the informed. On date 2 he makes a profit on all uninformed traders and again loses to the informed. On date 1 the market maker is the only liquidity provider. On date 2 the market maker will be the liquidity provider unless there are informed or uninformed limit orders in the book, in which case these take precedence over him. His role is to provide liquidity to traders in case no other traders on the opposite side submitted a limit order.

In equilibrium the specialist quote-updating rule, the informed trader's order submission strategy, and the patient uninformed trader's limit price-setting rule are determined jointly.

DEFINITION 1. An equilibrium is defined by an order submission rule for the informed traders, a limit price-setting rule for the patient uninformed traders, and a quote-updating rule for the market maker such that

1. the quote-updating rule satisfies $a_1 = E[v|MB]$, $b_1 = E[v|MS]$, $b_2(X) = E[v|X, MS]$, and $a_2(X) = E[v|X, MB]$, for any $X \in O$, which is the order received in the first period;
2. the informed trader's order submission rule maximizes expected profits; and
3. the limit price-setting rule of the patient uninformed traders minimizes (maximizes) the expected purchasing (selling) price.

Since the asset value is continuously distributed, we restrict our analysis to quote-updating rules of the market maker that are continuous in the limit price of a limit order. As the proposition below demonstrates, within the context of our model, this implies that in an equilibrium all traders who submit limit orders post them with the same limit price.

PROPOSITION 1. Assume that the quote-updating rule of the market maker is continuous in the limit price. In equilibrium all limit buy (sell) orders are posted at a price PB (PS) such that $b_2(LB;\ PB) = PB$ ($a_2(LS;\ PS) = PS$).[5]

Proof. See Appendix A.

As the proposition shows, the time 1 limit buy (sell) price must be the same as the time 2 bid (ask). The reason is that there is going to be at most only one market order on the opposite side at time 2, and, as mentioned above, the time priority of orders is enforced and the market maker is assumed to yield to limit orders. Submitting a limit order at time 1 with a limit price that is strictly inside the time 2 spread is obviously suboptimal, since conditional on being executed at time 2, such a buy (sell) order would be executed at a worse price than the time 2 prevailing bid (ask). However, as we will show, the equilibrium limit order price is inside the time 1 spread.[6] It is important to keep in mind that both the informed traders and the patient uninformed traders know the market maker's quote-updating rule and as such can compute on their own what the market maker will post at time 2 conditional on the order submitted at time 1.

Since in equilibrium $b_2(LB;\ PB) = PB$ ($a_2(LS;\ PS) = PS$), for notational convenience we restrict the analysis to the case in which the market maker's updating rule is a function of the order type only without loss of generality. Thus $b_2(LB;\ PB) \equiv b_2(LB) = E[v|LB, MS]$ and $a_2(LS;\ PS) \equiv a_2(LS) = E[v|LS, MB]$.

Given an opportunity to trade on date 1, an informed trader's expected profit is given by[7]

$$\pi_1 = \begin{cases} v - a_1 & \text{if places } MB \\ \frac{1}{2}\mu p[v - b_2(LB)] & \text{if places } LB \\ \frac{1}{2}\mu p[a_2(LS) - v] & \text{if places } LS \\ b_1 - v & \text{if places } MS. \end{cases}$$

The above expression implies that an informed trader will optimally place a buy (sell) order whenever $v \geq \max[m, b_2(LB)]$ ($v \leq \min[m, a_2(LS)]$). Since $b_2(LB)$ ($a_2(LS)$) may potentially be greater (smaller) than m, for some values of v the informed trader may opt not to trade at all on date 1.

On date 1 the informed trader will prefer placing a market buy over a limit

5. Relaxing the assumption of continuous quote-updating rules maintains the result that all traders placing a limit buy (sell) order will place it at the same price. However, that price might satisfy $b_2(LB;\ PB) < PB$ ($a_2(LS;\ PS) > PS$).

6. This is shown in lemma 5, which is embedded within the proof of proposition 2.

7. Note that the relative weight of the patient uninformed traders feeds into the informed traders' expected profits through the impact of l on a_1, b_1, $a_2(LS)$, and $b_2(LB)$.

buy if $\frac{1}{2}\mu p[v - b_2(LB)] \leq v - a_1$ or equivalently if

$$v \geq c \equiv \frac{a_1 - \frac{1}{2}\mu p b_2(LB)}{1 - \frac{1}{2}\mu p}. \tag{1}$$

In order to proceed, we first conjecture that in equilibrium $\bar{v} > c > a_1 > \max[m, b_2(LB)] > \underline{v}$ and later show that our conjecture in fact holds in equilibrium.

Since $c > a_1 > \max[m, b_2(LB)]$, an informed trader will place a limit buy order if and only if $c > v \geq \max[m, b_2(LB)]$. Specifically, when the value of the asset is above the ask (a_1) and below c, an informed trader will place a limit buy order. For such a trader the benefit of obtaining a potential lower price outweighs the execution risk of a limit order, and she will place a limit order even though the asset value is above the ask. However, if the value of the asset is above c, the benefit of sure execution dominates.

On the one hand, on date 1,

$$\begin{aligned} \Pr[MB|v] &= \tfrac{1}{2}\mu(1-l) + (1-\mu)1_{\{v \geq \max(c,a_1)\}}, \\ \Pr[LB|v] &= \tfrac{1}{2}\mu l + (1-\mu)1_{\{c > v \geq \max[m, b_2(LB)]\}}. \end{aligned} \tag{2}$$

On the other hand, on date 2 conditional on a limit buy on date 1,

$$\Pr[MS|LB, v] = \tfrac{1}{2}\mu + (1-\mu)1_{\{v < b_2\}}, \tag{3}$$

and applying Bayes' rule yields

$$\begin{aligned} a_1 &= E[v|MB] = \frac{\int_{\underline{v}}^{\bar{v}} x g(x) \Pr[MB|x] dx}{\int_{\underline{v}}^{\bar{v}} g(x) \Pr[MB|x] dx}, \\ b_2(LB) &= E[v|LB, MS] = \frac{\int_{\underline{v}}^{\bar{v}} x g(x) \Pr[LB|x] \Pr[MS|LB, x] dx}{\int_{\underline{v}}^{\bar{v}} g(x) \Pr[LB|x] \Pr[MS|LB, x] dx}. \end{aligned} \tag{4}$$

Combining equations (2), (3), and (4) yields that in equilibrium the following holds:

$$\frac{\mu}{2(1-\mu)}(1-l)(m-a_1) + \int_{\max(c,a_1)}^{\bar{v}} (x - a_1) g(x) dx = 0 \tag{5}$$

and

$$\frac{\mu l}{2(1-\mu)}[m - b_2(LB)] + \left\{\int_{\max[m, b_2(LB)]}^{c} [x - b_2(LB)]g(x)dx\right.$$
$$\left. + \, l\int_{\underline{v}}^{b_2(LB)} [x - b_2(LB)]g(x)dx\right\} = 0. \tag{6}$$

Furthermore, the participation constraint of the patient uninformed traders requires that

$$(1-p)m + \tfrac{1}{2}p\mu[b_2(LB) + m]$$
$$+ \, p(1-\mu)\left[\int_{\underline{v}}^{b_2(LB)} b_2(LB)g(x)dx + \int_{b_2(LB)}^{\bar{v}} xg(x)dx\right] < a_1, \tag{7}$$

where the left-hand side represents the expected transaction price from submitting a limit order and the right-hand side represents the transaction price from submitting a market order.

By construction, any solution of (5) and (6) that also satisfies (7) will be an equilibrium. The following proposition provides conditions for the existence of such an equilibrium.

PROPOSITION 2. If $(\mu, l, p) \in (0, 1)^3$ and $\int_0^{\bar{v}} xg(x)dx < \infty$, then an equilibrium exists.

Proof. See Appendix A.

Proposition 2 is dependent on our conjecture that $\bar{v} > c > a_1 > \max[m, b_2(LB)] > \underline{v}$. The following lemma demonstrates that our conjecture indeed holds.

LEMMA 1. In equilibrium, $\bar{v} > c > a_1 > \max[m, b_2(LB)] > \underline{v}$.

Proof. See Appendix A.

While Rock (1990) has conjectured that in equilibrium the informed traders will use only market orders, combining lemma 1 with the existence of an equilibrium implies that informed traders may in fact use both market and limit orders in equilibrium; as discussed earlier, informed traders may use limit orders even when the asset value is outside the bid-ask spread. It is worthwhile to note that the restrictions we impose on the distribution of the asset values are not critical for these results and are mainly for expositional purposes. Specifically, the assumption of symmetry of the density function around the mean is imposed simply to enable us to characterize the equilibrium by looking at only one side of the market and is not required to obtain the results.

We next demonstrate that not only do informed traders use limit orders, but also there exist equilibria in which a limit order conveys more information than a market order.

THEOREM 1. For any pair $(l, p) \in (0, 1)^2$, there exists $\mu^*(l, p) \in (0, 1)$ such that in equilibrium

1. Pr(limit order submission | informed trader) > Pr(limit order submission | uninformed trader) and
2. Pr(informed trader | limit order observed) > Pr(informed trader | market order observed).

Proof. See Appendix A.

Given the existence of equilibria in which limit orders convey more information, a natural question is when it is more likely that this happens. To answer this question we first analyze, within the context of our model, the major forces that induce informed traders to prefer limit orders over market orders.

We start by considering the following two properties of the optimal order submission strategy:

1. The probability of an informed trader using a limit order is an increasing function of the probability that the information is long lived.
2. The probability of an informed trader using a limit order is an increasing function of the probability that an uninformed trader places a limit order.

LEMMA 2. If $m > b_2(LB)$, then the above two properties hold.

Proof. See Appendix A.

PROPOSITION 3.

1. For any density function g, there exists an $l^* < 1$ such that, for $l > l^*$, the above two properties hold.
2. For any density function g and mass of patient uninformed traders l, there exists a $\mu^* < 1$ such that, for $\mu > \mu^*$, the above two properties hold.
3. For any density function g and mass of uninformed traders μ, there exists an $\tilde{l}$ such that, for $l < \tilde{l}$, the second property holds.
4. For the uniform distribution, the above two properties hold.

Proof. See Appendix A.

In addition, we verified numerically that if g is normally distributed, then the above two properties hold for a wide range of the parameter values.

All else equal, the longer the horizon of the information, the higher the probability that an informed trader would prefer placing a limit order and thereby bear execution risk in exchange for a better price. As a result, in an environment in which the fraction of uninformed traders who decide to place limit orders is relatively insensitive to the horizon of the potential information,

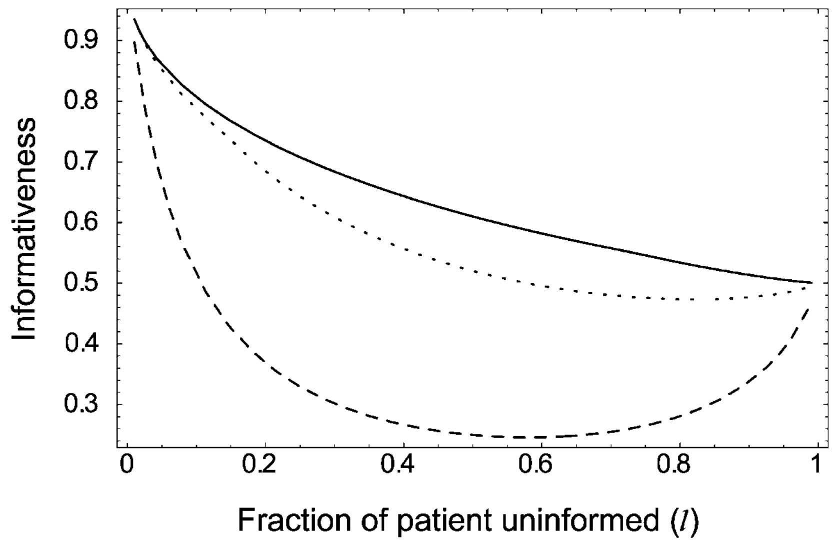

FIG. 2.—Informativeness of a limit order as a function of the fraction of uninformed traders that are patient (l) in the economy. The solid, dotted, and dashed lines are for $\mu = 0.1$, $\mu = 0.5$, and $\mu = 0.9$, respectively; $p = 0.5$.

longer-lived information should be associated with a greater probability that limit orders convey more information than market orders.[8]

The second part of the proposition is quite intuitive: When there are more uninformed traders using limit orders, it is easier for informed traders to use limit orders without being identified, and the execution probability is unchanged because all traders submit market orders at time 2 since the asset value will become public before time 3.[9]

Given the second part of proposition 2, one might be tempted to conclude that the probability that limit orders convey more information than market orders should also increase as the fraction of uninformed traders who place limit orders increases. In figure 2, we use the probability that a limit order is submitted by an informed trader to measure the informativeness of a limit order (Pr(informed trader | limit order observed)).[10] This figure shows that

8. Of course, in our model the fraction of patient uninformed traders is exogenous. In a full-equilibrium model, that fraction would be endogenous. In that case, a longer information horizon may correspond to a larger fraction of the uninformed traders submitting limit orders. This may affect the robustness of the above result.

9. While we have been able to prove this result only under the parameter restrictions imposed in the proposition, our conjecture is that it is true for a considerably wider range. We have numerically checked this issue for a wide range of parameter combinations; in all cases it was true for all values of l.

10. In our setting it is easy to see that Pr(informed trader | limit order observed) > 0.5 if and only if Pr(informed trader | limit order observed) > Pr(informed trader | market order observed).

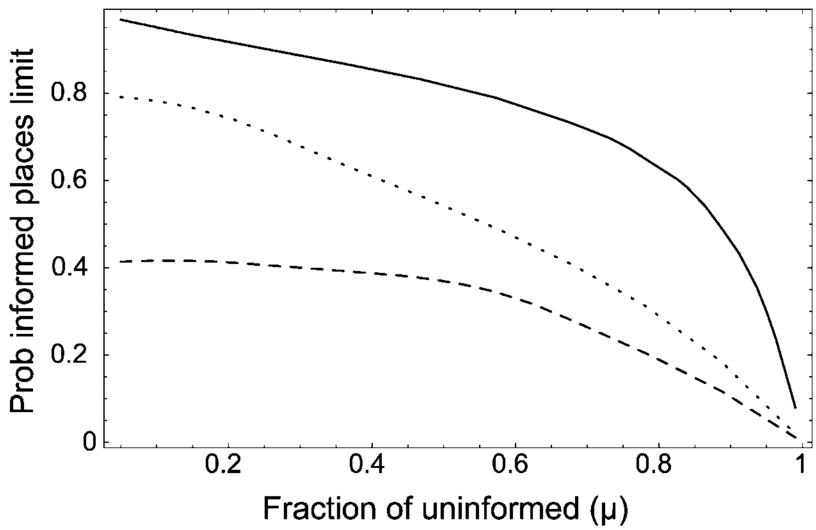

FIG. 3.—Probability of an informed trader placing a limit order as a function of the mass of uninformed (μ) in the economy. The solid, dotted, and dashed lines are for $l = 0.9$, $l = 0.5$, and $l = 0.1$, respectively; $p = 0.5$.

throughout most of the range, the informativeness of limit orders is in fact decreasing as a function of the mass of patient uninformed traders (l).[11] This implies that generally an increase in the fraction of patient uninformed traders, with the mass of uninformed traders overall held fixed, decreases the probability that a limit order is information driven. The reason is that as l increases, although the probability that an informed trader submits a limit order increases (which can be inferred from fig. 3), this probability does not increase as much as l. Intuitively, the potential benefit of an informed trader hiding inside a crowd of patient uninformed traders is lower than the benefit of hiding inside a crowd of impatient ones because of the participation constraint of the patient uninformed traders.

When both the overall mass of uninformed traders and the fraction of patient uninformed traders are high, limit order informativeness becomes an increasing function of the mass of patient traders (see, e.g., $\mu = 0.9$ and $l > 0.6$). This result is driven by the fact that a decrease in the mass of impatient uninformed traders increases the bid-ask spread, so that there are more realizations of v that will lie inside the spread. As the fraction of the patient uninformed traders approaches one, the bid-ask spread tends to infinity, because the specialist knows that the only traders who will potentially post

11. All the figures are plotted for the case in which the asset value is normally distributed with mean zero and variance one. The results are qualitatively similar for other variance values, as well as for a uniformly distributed asset value.

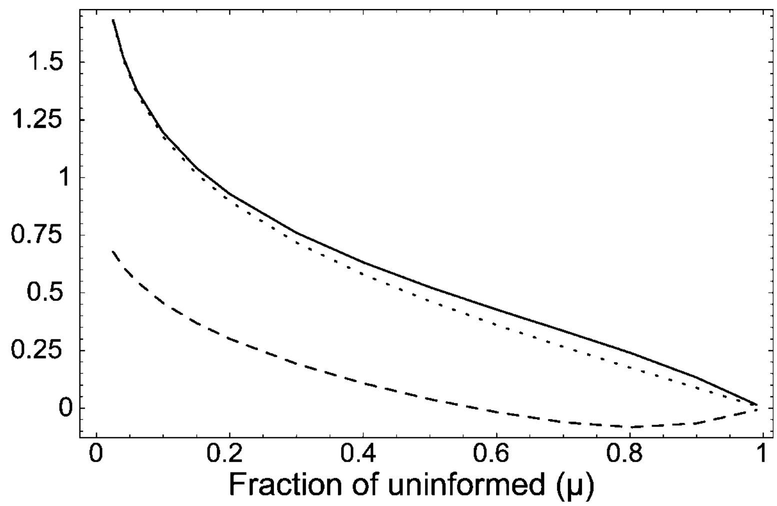

FIG. 4.—Time 1 ask price (a_1, dotted line), time 2 bid price ($b_2(LB)$, dashed line), and the threshold asset value for submitting a market order (c, solid line) plotted as functions of the mass of uninformed (μ) in the economy. $p = 0.5$ and $l = 0.1$.

market orders are informed ones. The infinite spread forces the informed traders to place only limit orders, if at all, so that the informativeness measure becomes 0.5.

Figure 3 shows that the ex ante probability that an informed trader places a limit order decreases as the fraction of uninformed traders in the economy increases, and therefore so does the probability that limit orders convey more information.[12] Intuitively, an increase in the overall proportion of uninformed traders in the economy has two effects on the informed trader's decision on whether to post a limit or a market order. First, it increases the probability that a limit order will be hit before the information is revealed. Therefore, the nonexecution risk of a limit order decreases. Second, the existence of more uninformed traders (or equivalently less informed traders) induces the specialist to decrease the time 1 ask price (a_1) as a result of the reduction in the adverse selection, and thus a market order becomes more attractive. Accordingly, the price improvement of a limit order over a market order may decrease. The ex ante probability that an informed trader uses a limit order depends on the relative profitability of a limit order over a market order, measured by the difference between the expected profit from a limit order and the profit from a market order. This measure takes both of the above-mentioned effects into account. As shown in figure 4, the time 2 limit price indeed decreases over most of the range, which implies that, conditional on

12. We verified numerically that the second effect also dominates for many other asset value distribution functions (e.g., a uniform distribution).

execution, the profit from submitting a limit order increases. Thus, consistent with Glosten (1994) and Handa and Schwartz (1996), less informed traders (equivalently more uninformed traders) make a limit order more profitable given that the limit order execution probability also increases. However, since the decision on the order type choice depends on the relative profitability of a limit order over a market order, one needs also to examine how the profit of a market order changes as the fraction of uninformed traders increases. Figure 4 shows that the time 1 ask price a_1 decreases significantly as a result of the presence of less informed traders and thus less adverse selection. This implies that the profit from submitting a market order also increases as the fraction of uninformed traders increases. In addition, figure 4 also shows that the threshold value c (i.e., the asset value v above which it is optimal to submit a market order) also decreases. This implies that for any fixed asset value v, the profit from submitting a market order increases more rapidly than the expected profit from submitting a limit order ($\frac{1}{2}\mu p[v - b_2(LB)]$) because of the nonexecution risk of a limit order. Consequently, even though the profitability of a limit order is improved, the *relative* profitability of a limit order decreases. Therefore, the probability of submitting a limit order decreases as the fraction of uninformed trader increases as shown in figure 3.

As noted in proposition 3, as the information becomes longer lived, the execution risk that a trader bears from placing a limit order decreases, thereby inducing more informed traders to choose a limit order over a market order. As a result, the proportion of market orders that are placed by informed traders declines. Since the market maker loses when trading against informed traders, a decrease in the proportion of informed market orders implies a decrease in the bid-ask spread at the initial date, as shown in the first part of the following proposition.

PROPOSITION 4.

1. The ask price on the initial date is a decreasing function of the probability that the information is long lived.
2. The ask price on the initial date is an increasing function of the probability that an uninformed trader places a limit order.

Proof. See Appendix A.

To understand the second part of the proposition, recall that figure 2 shows that the informativeness of limit orders generally decreases as the probability of uninformed traders placing a limit order increases. This in turn implies that the proportion of market orders that convey information increases, resulting in an increase in the bid-ask spread.

To briefly summarize, in addition to demonstrating the existence of equilibria in which limit orders are more likely than market orders to be information driven, our model has the following implications: (1) longer-lived information tends to increase the relative informativeness of limit orders versus market orders; (2) longer-lived information decreases the bid-ask spread; (3) an increase in the fraction of uninformed traders decreases (increases) the probability of limit (market) orders being informative; (4) if uninformed traders are predominantly impatient (patient), then an increase in the fraction of patient uninformed traders tends to decrease (increase) the relative informativeness of limit orders versus market orders; and (5) an increase in the fraction of uninformed patient traders (with the overall proportion of uninformed traders held fixed) increases the spread.

III. Who Uses Limit Orders?

Our theoretical model implies that under some conditions limit orders can be more informative than market orders, and the reverse could be true in other cases. In this section, we use the TORQ database to examine whether limit or market orders are more informative in the NYSE. In addition, we also investigate whether the specialists' perceptions of the informativeness of the two order types are consistent with these orders' actual informativeness.

Previous empirical research has evaluated the influence of different factors on the quote-updating process. Somewhat surprisingly, the relative impact of market versus limit orders has been largely ignored. Specifically, no one has compared the informational content conveyed by market versus limit orders. Kavajecz and Odders-White (2001) find that changes in the best bid and ask in the limit order book can have a large impact on the posted price schedule. However, this is not a comparison between limit orders and market orders because market orders can also alter the best bid and ask in the limit order book.

Next, we briefly describe some relevant features of the data and the construction of event series for each of the securities in the TORQ data set.

A. Data and Construction of Event Series

The TORQ database covers 144 stocks from November 1, 1990, through January 31, 1991 (63 trading days). It includes all transactions, all orders submitted via the automated routing system, and all quote changes for these stocks. The 144 stocks include 15 stocks from each of the top four market cap deciles on the NYSE and 14 stocks from each of the lower six deciles.

The different tests we conduct require us to construct from the data set a detailed time series of events for each of the securities.[13] Each event series includes order submissions, quote revisions by the specialist, and transactions. Before conducting the tests, we discard any quote records that are not NYSE quotes (i.e., Intermarket Trading System [ITS] quotes). In many cases ITS quotes are auto quotes that just follow the NYSE quotes.

Two important steps for our tests are computing the sample frequencies of each order type and attributing specialists' quote revisions to different order types. When calculating the sample frequencies of market and limit orders,

13. A detailed description of the TORQ database and its different files is contained in Hasbrouck and Sosebee (1992).

TABLE 1 **Descriptive Statistics**

Decile	Observations	LIMIT	MKT	STP	BIDC	ASKC
Active	15	252.1	237.2	73.5	36.7	37.5
2	15	58.9	52.7	13.5	13.0	12.8
3	15	31.7	39.9	11.9	11.4	11.7
4	15	23.5	24.4	5.8	7.4	7.3
5	14	20.5	11.5	2.1	7.1	7.1
6	14	13.8	8.4	1.8	5.3	5.2
7	14	10.7	5.1	1.0	4.0	3.9
8	14	7.2	3.4	.5	2.9	2.9
9	14	4.6	2.4	.3	2.4	2.3
Inactive	14	1.7	1.1	.1	1.1	1.1
All	144	43.8	40.0	11.5	9.4	9.4

NOTE.—This table reports the average number per day of the following activities: limit orders (LIMIT), market/marketable limit orders (MKT), stopped orders (STP), bid changes (BIDC), and ask changes (ASKC). Stocks are sorted according to activity. Observations represents the number of stocks in a decile. Activity is defined as the total number (over all 63 trading days) of market + limit orders.

we use the following criteria. First, we count only orders that are straight market orders or standard limit orders.[14] We restrict the analysis to limit orders at the quote or better and treat marketable limit orders as market orders.[15] Straight market orders and standard limit orders account together for about 95% of the SuperDot orders. We do not count market-on-close orders or other orders with rarely used qualifications. A market-on-close order is an order to be executed only at the end of the day. A specialist's reaction to such an order is probably different from the specialist's reaction to a regular market order. Only about 2% of all orders are market-on-close orders.

Note that even when a certain event is not part of a test, it is *not* dropped from the time series of events. Although these activities are not counted as part of the relevant sample frequencies, they are part of a specialist's information set; excluding them might distort the results.

Trading activity.—The information revelation process of actively traded stocks may be different from that of inactively traded stocks. Furthermore, Hasbrouck and Sofianos (1993) find that the specialist's participation rate is about 19% for inactively traded stocks, whereas for actively traded stocks the participation rate drops to about 10%. In order to control for different trading activities, we sort stocks on the basis of trading activity and group them into deciles; we use the total number of market and limit orders, over all 63 trading days, as a measure of a stock's trading activity.

Table 1 gives some descriptive data about the stocks studied. The table records average daily frequencies of different events for each trading activity decile. The average measure of trading activity of the most active decile is more than 170 times that of the least active decile. In contrast, the average number of quote price changes of the most active decile is only about 33

14. Limit orders include only noncanceled limit orders. For robustness we also performed the analysis with all limit orders without considering cancelations. The results are similar and are available on request.

15. The results are similar if instead we exclude marketable limit orders from the analysis.

times that of the least active decile. This may suggest that orders convey more information or have a larger effect on specialists' inventory positions for inactive stocks than for active stocks.[16]

B. The Information Conveyed by Orders

Our objective in this subsection is to analyze whether informed traders tend to place more limit orders than market orders. Following Huang and Stoll (1996), we determine the *informativeness* of the different types of orders by comparing, across the two order types, the conditional probabilities of the quote midpoint being higher (lower) following a submission of a buy (sell) order than the level of the quote midpoint that was in place just before the order was submitted. One of course needs to decide how long after the order submission the measurement should be taken. There are no theoretical guidelines for the choice of the appropriate horizon. In our tests we have decided to use an hour and a day from submission since we intend to measure the longer-horizon impact of an order.

Definition informativeness of an order type at a one-hour/one-day horizon is measured as the conditional probability that the quote midpoint an hour/a day after submission of a buy (sell) order is higher (lower) than the quote midpoint that was in place just before the order was submitted.[17]

The tested hypotheses are as follows.

H_0. Market orders and limit orders are equally informative.

H_1. Market orders are more informative.

H_1'. Limit orders are more informative.

Thus, in comparisons of the informativeness of market buy (sell) orders to limit buy (sell) orders at the one-hour/one-day horizon, the null is that the conditional probability that the quote midpoint is above (below) the one prior to submission is the same across the two order types; H_1 (H_1') states that the conditional probability following a market (limit) buy order is higher than that following a limit (market) buy order.

Through most of the analysis we use a nonparametric test statistic that is similar to the one used in Rubinstein (1985) and is described in detail below. An advantage of using this procedure in our context is that it does not require specifying a priori variable relations. We also use a parametric probit analysis both as a robustness check and as a means for explicitly controlling for order size, relative frequency of trading (i.e., whether trading is faster or slower than typical trading for that stock), and other variables. Below we briefly describe our nonparametric approach. We postpone the description of the probit model until we introduce the probit regression results.

16. This finding is consistent with that in the study by Madhavan and Smidt (1991), who demonstrate that a trade in an active stock has a smaller impact than a corresponding trade in a less active stock.

17. For example, informativeness of market buys at a one-day horizon is defined as the ratio of the number of times the quote midpoint a day after a submission of a market buy is higher than the one prior to the submission to the number of market buy order submissions.

The hypothesis-testing procedure is as follows. First, we denote by P_{mkt} $(1 - P_{\mathrm{mkt}})$ the probability that a submitted order is a market order (limit order). This probability is obtained by using the information in table 1 to compute the sample frequencies of market orders and limit orders. Specifically, P_{mkt} is the fraction of submitted orders that are market orders. Let n be the total number of times the quote midpoint after an hour (a day) is in the *correct direction* (i.e., above the one at submission for a buy order and below the one at submission for a sell order) following a submission of either a market order or a limit order.[18] Let n_{mkt} be the number of correct direction quote midpoint changes that follow market orders (so that $n - n_{\mathrm{mkt}}$ is the number of correct direction quote midpoint changes that follow limit orders). Under the null hypothesis H_0, the probability that out of these n quote revisions n_{mkt} or more are preceded by a market order is well approximated by

$$1 - N\left[\frac{n_{\mathrm{mkt}} - n \cdot P_{\mathrm{mkt}}}{\sqrt{n \cdot P_{\mathrm{mkt}} \cdot (1 - P_{\mathrm{mkt}})}}\right],$$

where N is the standard normal cumulative distribution function.

Performing this test using the TORQ database is straightforward. If this probability is lower (higher) than 0.05 (0.95), we reject the null of equal informativeness in favor of the alternative H_1 (H_1') that market (limit) orders are more informative.

Tables 2 and 3 report the results for a one-hour horizon and a one-day horizon, respectively. In both tables, panel A compares market orders that are executed at the quote versus limit orders that are submitted at the quote, and panel B compares market orders that get a price improvement versus limit orders that are inside the quote. Sell orders and buy orders are analyzed separately. Order size relative to the quoted depth is partitioned into three categories: (1) small, less than half the quoted depth; (2) medium, greater than or equal to half the quoted depth but less than the quoted depth; and (3) large, greater than or equal to the quoted depth. The relevant depth used for a market buy (sell) is the ask (bid) depth, and for a limit buy (sell) it is the bid (ask) depth.[19]

For small and medium orders the results support the hypothesis that limit orders are more informative. At the one-hour horizon, for small orders the null of equal informativeness is rejected at the 1% level against the alternative of limit orders being more informative for almost all cases, both in panel A and in panel B. For medium orders, in panel A the majority are significant at least at 10%, and in panel B the null is rejected in the vast majority of cases at least at 2%. At the one-day horizon, in panel A, for both small and medium orders, in almost all cases the null is rejected against the alternative of limit orders being more informative at least at 5% and in many of these

18. Note that, in computing P_{mkt} $(1 - P_{\mathrm{mkt}})$, we account for cases of both *correct direction* and *incorrect direction* so that the test statistic takes full account of both.

19. The results are similar when the limit buy (sell) is compared to the ask (bid) depth.

TABLE 2 Relative Informativeness of Order Types: One-Hour Horizon

Decile	Sell Orders			Buy Orders		
	Small	Medium	Large	Small	Medium	Large
	A. At the Quote					
Active	1.38	1.52	.97	1.50	1.47	.99
	{>.99}	{>.99}	{.33}	{>.99}	{>.99}	{.43}
	(.40; .29; 24,236)	(.39; .26; 2,229)	(.40; .41; 3,058)	(.43; .29; 25,794)	(.45; .30; ,2538)	(.46; .47; 3,513)
2	1.32	1.54	1.03	1.19	1.19	.92
	{>.99}	{>.99}	{.63}	{>.99}	{.94}	{.18}
	(.34; .25; 4,646)	(.34; .22; 641)	(.37; .36; 1,269)	(.35; .29; 5,822)	(.34; .29; 692)	(.39; .43; 1,381)
3	1.16	1.21	.90	1.09	1.12	.94
	{>.99}	{.95}	{.12}	{.99}	{.85}	{.26}
	(.36; .31; 2,450)	(.32; .27; 504)	(.38; .42; 1,117)	(.37; .34; 2,660)	(.39; .35; 484)	(.40; .43; 982)
4	1.23	1.13	.96	.77	1.03	.80
	{>.99}	{.81}	{.34}	{<.01}	{.58}	{.01}
	(.20; .16; 1,226)	(.28; .25; 259)	(.30; .32; 642)	(.20; .26; 1,786)	(.30; .29; 269)	(.31; .40; 691)
5	1.36	1.21	1.11	1.28	1.39	.89
	{>.99}	{.90}	{.82}	{>.99}	{.97}	{.15}
	(.31; .23; 857)	(.31; .25; 219)	(.35; .31; 665)	(.32; .25; 1,020)	(.32; .23; 283)	(.34; .38; 768)
6	1.12	1.05	.96	1.36	1.56	.93
	{.90}	{.61}	{.35}	{>.99}	{.98}	{.31}
	(.24; .21; 490)	(.27; .26; 156)	(.33; .35; 471)	(.25; .19; 549)	(.33; .21; 171)	(.34; .37; 498)
7	1.73	1.58	.89	1.21	1.82	.74
	{>.99}	{.94}	{.24}	{.97}	{.99}	{.02}
	(.22; .13; 278)	(.24; .15; 93)	(.28; .32; 326)	(.25; .21; 434)	(.31; .17; 136)	(.31; .42; 395)
8	1.80	.84	.95	1.21	1.10	.87
	{>.99}	{.22}	{.39}	{.92}	{.64}	{.25}
	(.24; .13; 188)	(.26; .31; 90)	(.30; .31; 271)	(.26; .22; 250)	(.32; .29; 102)	(.29; .33; 271)

9	1.37	1.59	.69	2.43	2.32	.77
	{.94}	{NA}	{.03}	{>.99}	{NA}	{.10}
	(.19; .14; 105)	(.26; .16; 40)	(.22; .31; 148)	(.29; .12; 107)	(.29; .13; 48)	(.33; .43; 242)
Inactive	1.01	.73	1.02	1.13	·	.91
	{NA}	{NA}	{.52}	{NA}	{NA}	{.42}
	(.12; .12; 29)	(.18; .25; 10)	(.31; .30; 73)	(.22; .19; 30)	(.17; .00; 8)	(.28; .31; 91)
	B. Better than the Quote					
Active	1.41	1.60	.95	1.53	1.37	.91
	{>.99}	{>.99}	{.25}	{>.99}	{>.99}	{.09}
	(.42; .30; 20,326)	(.41; .25; 1712)	(.37; .39; 1,372)	(.46; .30; 22,925)	(.46; .34; 1,901)	(.44; .49; 1,691)
2	1.35	1.64	.92	1.19	1.29	.78
	{>.99}	{>.99}	{.23}	{>.99}	{.98}	{.01}
	(.38; .28; 4,615)	(.39; .24; 573)	(.36; .40; 707)	(.40; .33; 5,830)	(.39; .30; 615)	(.40; .52; 796)
3	1.23	1.47	1.11	1.22	1.20	.95
	{>.99}	{>.99}	{.84}	{>.99}	{.95}	{.32}
	(.42; .34; 2,916)	(.38; .26; 500)	(.43; .39; 830)	(.44; .36; 2,954)	(.44; .37; 488)	(.42; .44; 648)
4	1.24	1.40	1.05	.94	1.20	.98
	{>.99}	{.98}	{.64}	{.08}	{.90}	{.43}
	(.26; .21; 1,391)	(.34; .24; 261)	(.37; .35; 486)	(.29; .31; 1,988)	(.38; .32; 307)	(.44; .45; 524)
5	1.65	1.66	1.20	1.44	1.64	.92
	{>.99}	{>.99}	{.90}	{>.99}	{>.99}	{.24}
	(.40; .24; 990)	(.41; .25; 245)	(.41; .34; 452)	(.42; .29; 1,241)	(.43; .26; 341)	(.43; .47; 584)
6	1.31	1.43	1.07	1.67	2.99	1.08
	{>.99}	{.98}	{.70}	{>.99}	{>.99}	{.69}
	(.34; .26; 614)	(.36; .25; 172)	(.46; .43; 412)	(.36; .21; 690)	(.44; .15; 214)	(.45; .42; 434)
7	2.25	1.85	1.11	1.25	2.31	.75
	{>.99}	{.99}	{.70}	{.99}	{>.99}	{.04}
	(.33; .15; 361)	(.37; .20; 121)	(.38; .35; 251)	(.37; .30; 573)	(.39; .17; 143)	(.38; .51; 308)
8	2.50	1.12	1.00	1.87	1.77	.87
	{>.99}	{.70}	{.50}	{>.99}	{.94}	{.24}
	(.38; .15; 243)	(.40; .36; 114)	(.39; .39; 214)	(.42; .22; 317)	(.42; .24; 108)	(.42; .48; 228)
9	2.01	1.75	1.67	2.69	2.87	1.41
	{>.99}	{NA}	{.97}	{>.99}	{.98}	{.89}

TABLE 2 ***(Continued)***

Decile	Sell Orders Small	Sell Orders Medium	Sell Orders Large	Buy Orders Small	Buy Orders Medium	Buy Orders Large
	(.36; .18; 140)	(.30; .17; 42)	(.44; .26; 176)	(.40; .15; 147)	(.44; .15; 71)	(.53; .38; 226)
Inactive	1.63	3.00	.86	2.16	1.19	1.18
	{NA}	{NA}	{.37}	{NA}	{NA}	{.61}
	(.24; .15; 35)	(.38; .13; 13)	(.37; .43; 53)	(.32; .15; 39)	(.30; .25; 13)	(.39; .33; 65)

NOTE.—This table presents the ratio of the conditional probability of an increase (decrease) in the quote midpoint(relative to its level immediately before the submission of an order) one hour after the submission given a limit buy (sell) to the corresponding conditional probability of an increase (decrease) given a submission of a market buy (sell), while controlling for order size relative to the quoted depth. Panel A compares market orders that are executed at the quote vs. limit orders that are submitted at the quote, and panel B compares market orders that get a price improvement vs. limit orders that are inside the quote. Order size relative to depth is partitioned into the following three categories: (1) small, less than half the quoted depth; (2) medium, greater than or equal to half the quoted depth but less than the quoted depth; and (3) large, greater than the quoted depth. Stocks are sorted according to activity (15 in each of the first four deciles and 14 in each of the last six), where activity is defined as the total number (over all 63 trading days) of market orders. The probability of the sample event under the null against the alternative of market orders being more informative is presented in braces. In parentheses, from left to right, are the conditional probability of the quote midpoint increasing (decreasing) given a submission of a limit buy (sell), the conditional probability of the quote midpoint increasing (decreasing) given a submission of a market buy (sell), and the number of observations in the corresponding test. An NA appears if the number of observations for a test is less than 50. A · appears if the denominator of a division is 0.

TABLE 3 Relative Informativeness of Order Types for a One-Day Horizon

	Sell Orders			Buy Orders		
Decile	Small	Medium	Large	Small	Medium	Large
			A. At the Quote			
Active	1.30 {>.99} (.43; .33; 26,418)	1.45 {>.99} (.43; .29; 2,449)	.93 {.12} (.44; .47; 3,354)	1.46 {>.99} (.52; .36; 31,794)	1.58 {>.99} (.53; .34; 3,005)	1.01 {.55} (.52; .52; 3,954)
2	1.15 {>.99} (.44; .38; 6,365)	1.30 {.99} (.38; .30; 731)	1.05 {.71} (.44; .41; 1,495)	1.17 {>.99} (.49; .42; 8,303)	1.57 {>.99} (.49; .31; 964)	1.06 {.76} (.50; .47; 1,722)
3	1.15 {>.99} (.44; .38; 2,995)	1.31 {.99} (.41; .31; 635)	.96 {.32} (.41; .43; 1,216)	1.11 {>.99} (.56; .51; 4,037)	1.27 {.99} (.55; .44; 668)	1.03 {.65} (.53; .52; 1,292)
4	1.29 {>.99} (.32; .25; 1,967)	1.34 {.98} (.37; .28; 327)	.94 {.26} (.40; .43; 851)	.92 {.01} (.38; .41; 3,095)	1.23 {.96} (.48; .39; 417)	.96 {.35} (.46; .47; 971)
5	1.12 {.98} (.43; .39; 1,327)	1.14 {.84} (.41; .36; 295)	1.05 {.69} (.40; .38; 783)	1.13 {.99} (.47; .42; 1,571)	1.53 {>.99} (.47; .31; 413)	1.07 {.73} (.46; .43; 1,013)
6	1.03 {.66} (.35; .34; 763)	1.17 {.87} (.41; .35; 227)	1.18 {.92} (.42; .36; 580)	1.18 {>.99} (.46; .39; 1,057)	1.24 {.90} (.47; .38; 256)	.98 {.42} (.49; .50; 711)
7	1.29 {>.99} (.36; .28; 501)	1.34 {.91} (.38; .28; 150)	1.19 {.87} (.39; .33; 443)	1.17 {.98} (.47; .40; 823)	1.30 {.90} (.41; .32; 189)	.98 {.45} (.45; .46; 559)
8	1.49 {>.99} (.41; .27; 352)	1.13 {.73} (.38; .33; 120)	.99 {.47} (.37; .37; 335)	1.24 {.98} (.45; .36; 419)	1.65 {.97} (.49; .29; 145)	.94 {.36} (.43; .46; 408)

TABLE 3 ***(Continued)***

	Sell Orders			Buy Orders		
Decile	Small	Medium	Large	Small	Medium	Large
9	1.27 {.96} (.40; .32; 236)	1.57 {.92} (.36; .23; 57)	.79 {.07} (.36; .46; 242)	1.68 {>.99} (.41; .24; 163)	3.32 {.99} (.41; .13; 67)	.80 {.10} (.45; .57; 326)
Inactive	.74 {.12} (.24; .33; 72)	1.00 {NA} (.25; .25; 13)	1.59 {.85} (.40; .25; 92)	.97 {.47} (.37; .38; 53)	1.83 {NA} (.46; .25; 24)	1.53 {.83} (.48; .31; 150)
	B. Better than the Quote					
Active	1.20 {>.99} (.43; .36; 16,063)	1.50 {>.99} (.43; .29; 1,582)	.69 {<.01} (.42; .61; 6,478)	1.46 {>.99} (.56; .38; 20,012)	1.68 {>.99} (.55; .33; 1,856)	1.02 {.57} (.54; .53; 8,269)
2	1.10 {.99} (.42; .38; 3,572)	1.48 {.98} (.40; .27; 530)	1.00 {.50} (.43; .43; 1,899)	1.13 {>.99} (.52; .46; 4,485)	1.94 {>.99} (.54; .28; 642)	.89 {.20} (.50; .56; 2,002)
3	1.18 {>.99} (.44; .37; 2,329)	1.85 {>.99} (.43; .23; 451)	.87 {.11} (.41; .47; 1,441)	1.08 {.97} (.56; .52; 2,662)	1.56 {>.99} (.56; .36; 467)	1.08 {.77} (.55; .51; 1,561)
4	1.01 {.56} (.41; .41; 1,183)	1.62 {.96} (.39; .24; 202)	1.16 {.80} (.43; .37; 838)	1.13 {.99} (.50; .45; 1,462)	1.72 {.99} (.52; .30; 296)	.77 {.03} (.48; .63; 947)
5	1.40 {>.99} (.46; .33; 772)	1.49 {.94} (.47; .31; 216)	1.16 {.73} (.40; .35; 633)	1.36 {>.99} (.51; .37; 1,047)	1.61 {.92} (.50; .31; 313)	1.18 {.72} (.47; .40; 973)
6	1.02 {.60} (.42; .41; 438)	1.04 {.56} (.41; .39; 128)	1.19 {.76} (.42; .35; 503)	1.02 {.60} (.50; .49; 735)	1.41 {.90} (.51; .36; 214)	1.35 {.86} (.52; .38; 799)
7	1.13 {.80} (.46; .41; 342)	1.75 {.90} (.44; .25; 101)	1.37 {.84} (.46; .33; 415)	1.28 {.97} (.53; .41; 487)	1.45 {.86} (.47; .32; 131)	1.18 {.70} (.45; .38; 516)

8	1.52	3.77	.80	1.16	1.97	.74
	{.99}	{.92}	{.29}	{.75}	{.88}	{.20}
	(.45; .29; 195)	(.47; .13; 74)	(.37; .46; 275)	(.48; .41; 267)	(.49; .25; 105)	(.42; .57; 350)
9	1.58	1.92	1.81	1.03	·	1.12
	{.96}	{NA}	{.91}	{.53}	{NA}	{.61}
	(.43; .28; 89)	(.43; .22; 34)	(.43; .24; 200)	(.49; .47; 151)	(.55; .00; 56)	(.52; .46; 321)
Inactive	1.96	·	2.74	·	·	·
	{NA}	{NA}	{.85}	{NA}	{NA}	{.76}
	(.39; .20; 23)	(.38; .00; 9)	(.46; .17; 85)	(.39; .00; 30)	(.43; ·; 13)	(.51; .00; 116)

Note.—This table presents the ratio of the conditional probability of an increase (decrease) in the quote midpoint (relative to its level immediately before the submission of an order) one day after the submission given a limit buy (sell) to the corresponding conditional probability of an increase (decrease) given a submission of a market buy (sell), while controlling for order size relative to the quoted depth. Also see the note to table 2.

at 1% or 2%. In panel B, for the top five deciles the results strongly support limit orders being more informative both for small and for medium orders; in almost all cases the null is rejected at 1%. For the bottom five deciles the significance is less pronounced. However, if we pool these five deciles together, the null is rejected at 1% for both small and medium orders.

For large orders, at both the one-hour and the one-day horizons, there does not seem to be a statistically significant difference between the informativeness of limit orders and the informativeness of market orders in either panel.

Previous research has shown that information asymmetry is greatest at the beginning of the day (e.g., Madhavan, Richardson, and Roomans 1997). Furthermore, Bloomfield et al. (2003) provide experimental evidence suggesting that more private information and greater competition among informed traders in the morning make market orders more attractive to informed traders. In addition, toward the end of the trading day, specialists or other market participants potentially behave differently to control their overnight inventory exposure (see, e.g., Hong and Wang 2000). To help discern potential patterns in the behavior of informed traders throughout the day, we have also performed the analysis separately for the beginning (9:30 a.m.–10:30 a.m.), the middle (10:30 a.m.–3:00 p.m.), and the end (3:00 p.m.–4:00 p.m.) of the trading day. For all three time intervals the results are consistent with the ones reported: supporting the hypothesis that limit orders are more informative than market orders for small and medium orders, with no significant difference in informativeness for large orders.[20]

Our finding that limit orders actually contain more information contradicts the typical assumption made in the theoretical microstructure literature that informed investors prefer to use market orders whereas uninformed investors use limit orders.[21] This finding is consistent with the finding of Bloomfield et al. (2003), who also find that informed traders use more limit orders than liquidity traders do in an experimental asset market setting.

Intuitively, while a limit order entails execution risk, it has no price uncertainty, and conditional on execution, it constitutes a price improvement relative to using a market order. As our model demonstrates, an informed trader with long-lived information might prefer placing a limit order. To support the conjecture that informed traders tend to use market orders, most of the existing theoretical microstructure literature typically assumes implicitly that the information horizon is very short. Our empirical results suggest that this need not be the case.

The TORQ data set allows us to partially distinguish between orders placed by institutions and orders placed by individual investors. For a robustness check, we also ran the test separately for institutional orders. The results (not

20. Results are available from the authors. In order to have enough observations, we conducted this analysis after pooling together orders at the quote and those that are better than the quote.

21. One may argue that this assumption is made for model tractability and not as an ideological position. First, as we have pointed out, some authors have argued their positions on theoretical grounds quite forcefully. Second, it is not evident that this is an innocuous modeling assumption.

reported) are consistent with the results presented in the paper. Furthermore, comparing the results across the two samples (i.e., the one in the paper and the one in which we restrict to institutional orders) suggests that the relative informativeness of limit orders over market orders is greater for institutional orders than for orders placed by individuals. Furthermore, our results do not change if we exclude program trades. This is consistent with evidence in Hasbrouck (1996) that finds that there is no economically significant difference between the information conveyed by market orders that are program trades and market orders that are nonprogram trades.

While our nonparametric approach has an important advantage in that it does not require specifying a priori variable relations, it is less able to control for factors that might affect the choice of order type compared to a parametric analysis. For example, our nonparametric analysis splits order sizes into three categories relative to the quoted spread. However, this order size grid may be not fine enough. Given that there can be systematic differences in the sizes of limit orders versus market orders and that, as shown, for example, in Easley and O'Hara (1987), order size can be associated with information, we next conduct a probit analysis that explicitly controls for the size of an order. In addition, abnormal trading intensity may indicate the existence of information and may matter for the choice of a market versus a limit order. Other variables such as depth, spread, and previous returns can all affect the order type choice.[22]

Table 4 presents the results from a probit regression in which the dependant variable is whether the quote midpoint after an hour (in panel A) or a day (in panel B) is higher (lower) than the midpoint immediately before the submission of a buy (sell) order. The independent variables are a dummy variable (LIMIT) that takes the value of one if the order is a limit and zero if it is a market, the number of shares ordered (SHRNO), a standardized relative trading intensity variable (TRDINT), the prevailing spread (SPREAD), the prevailing ask (bid) depth (SDSIZE) for a buy (sell) order, the prevailing bid (ask) depth (ODSIZE) for a buy (sell) order, and the return in the previous interval multiplied by one for a buy order and by minus one for a sell order (ADJRET). We measure the relative trading intensity by computing the number of standard deviations away from the mean for each order. Specifically, this trading intensity variable is constructed as follows. First, following Madhavan et al. (1997), we divide each trading day into five periods: 9:30 a.m.–10:00 a.m., 10:00 a.m.–11:30 a.m., 11:30 a.m.–2:00 p.m., 2:00 p.m.–3:30 p.m., and 3:30 p.m.–4:00 p.m. Second, for each stock and each order we compute the number of transactions in the period within which the order was submitted. Third, for each stock and each period we compute the mean and the standard deviation of the number of transactions over the entire sample. Finally, for each stock and each order we subtract the average number of transactions in the specific interval within which the order was submitted from the number of transactions

22. We thank an anonymous referee for suggesting this complementary analysis.

TABLE 4 Relative Informativeness of Order Types for Probit Analysis

Decile	LIMIT	SHRNO	TRDINT	SPREAD	SDSIZE	ODSIZE	ADJRET
	A. One-Hour Horizon						
Active	.0788	.0017	.0376	.6859	−.0731	.0298	−.6390
	{<.01}	{.78}	{<.01}	{.78}	{<.01}	{<.01}	{<.01}
2	.0905	.0843	.0950	1.6911	−.1294	.0743	−2.7820
	{<.01}	{<.01}	{<.01}	{<.01}	{<.01}	{<.01}	{<.01}
3	.1051	.0335	.0982	1.1674	−.3921	.2055	−.9700
	{<.01}	{.06}	{<.01}	{.06}	{<.01}	{<.01}	{<.01}
4	.1158	.0593	.1453	1.9024	−.2084	.0335	−5.4303
	{<.01}	{.01}	{<.01}	{.01}	{<.01}	{<.01}	{<.01}
5	.2054	.1391	.1186	1.7177	−.3002	.1398	−3.1001
	{<.01}	{<.01}	{<.01}	{<.01}	{<.01}	{<.01}	{<.01}
6	.2514	.1242	.1046	1.7686	−.4674	.0925	−2.2698
	{<.01}	{<.01}	{<.01}	{<.01}	{<.01}	{<.01}	{<.01}
7	.2661	.0245	.1246	2.5607	−.2034	.0247	−.9273
	{<.01}	{.58}	{<.01}	{.58}	{<.01}	{<.01}	{<.01}
8	.2765	.0051	.1138	2.0538	−.3692	.0449	−.0355
	{<.01}	{.92}	{<.01}	{.92}	{<.01}	{<.01}	{<.01}
9	.3961	.0950	.1245	.7082	−.8146	.2265	1.0524
	{<.01}	{.28}	{<.01}	{.28}	{<.01}	{<.01}	{<.01}
Inactive	.5246	−.0182	.0425	.0865	−.1961	.0117	−.5514
	{<.01}	{.74}	{.08}	{.74}	{.08}	{.27}	{<.01}
	B. One-Day Horizon						
Active	.0250	−.0223	.0436	.1624	.0034	−.0030	2.0776
	{<.01}	{<.01}	{<.01}	{<.01}	{<.01}	{<.01}	{<.01}
2	.0263	.0102	−.0269	.6684	.0349	−.0274	−.1243
	{.01}	{.41}	{<.01}	{.41}	{<.01}	{<.01}	{<.01}
3	.0221	.0751	.0017	.2441	.1188	−.0880	−.1529
	{.10}	{<.01}	{.78}	{<.01}	{.78}	{<.01}	{<.01}
4	.0811	.0166	.0555	.9973	.0299	−.1304	−.2781
	{<.01}	{.46}	{<.01}	{.46}	{<.01}	{<.01}	{<.01}
5	.0449	.0040	.0294	.3637	.0514	−.1606	−.3992
	{.04}	{.88}	{<.01}	{.88}	{<.01}	{<.01}	{<.01}
6	.0749	.0129	.0509	.4856	.1234	−.4018	1.7001
	{<.01}	{.66}	{<.01}	{.66}	{<.01}	{<.01}	{<.01}
7	.1120	−.0274	.0024	1.1489	−.0034	−.1422	1.9076
	{<.01}	{.52}	{.83}	{.52}	{.83}	{<.01}	{.76}
8	.1281	−.0951	.0312	.7720	.0356	−.4031	−1.6877
	{<.01}	{.06}	{.02}	{.06}	{.02}	{<.01}	{.17}
9	.1752	.0978	−.0218	.4833	.1782	−.4402	.2560
	{<.01}	{.26}	{.20}	{.26}	{.20}	{<.01}	{<.01}
Inactive	.3787	.0396	.0556	.3097	.0374	−.2084	−.2780
	{<.01}	{.47}	{.02}	{.47}	{.02}	{.12}	{.12}

NOTE.—This table presents the coefficients from a probit analysis, where the dependant variable is equal to one if the quote midpoint after one hour (panel A) or one day (panel B) increased (decreased) relative to its level before the submission of a buy (sell) order, and zero otherwise. The independent variables are as follows. LIMIT takes the value of one (zero) if the order is a limit (market) order; SHRNO is the number of shares ordered; TRDINT represents the relative trading intensity for the time interval within which the order was submitted, in terms of the number of standard deviations from the sample average intensity for that time interval. For the purpose of computing TRDINT, the trading day is divided into the following intervals: 9:30 a.m.–10:00 a.m., 10:00 a.m.–11:30 a.m., 11:30 a.m.–2:00 p.m., 2:00 p.m.–3:30 p.m., and 3:30 p.m.–4:00 p.m. SPREAD is the prevailing spread. SDSIZE (ODSIZE) is the prevailing ask (bid) depth for a buy order and the prevailing bid (ask) depth for a sell order. ADJRET is the return in the previous hour (day) multiplied by one for a buy order and by minus one for a sell order. Stocks are sorted according to activity (15 in each of the first four deciles and 14 in each of the last six), where activity is defined as the total number (over all 63 trading days) of market + limit orders. *p*-values for the coefficients being zero are presented in braces.

in that period and then divide this difference by the sample standard deviation of the number of transactions in that interval.

Since in the nonparametric analysis there were no significant differences between the case in which we compared market orders that are executed at the quote versus limit orders that are submitted at the quote (panel A of tables 2 and 3) and the case in which we compared market orders that get a price improvement versus limit orders that are inside the quote (panel B of tables 2 and 3), we present the results of the probit analysis without splitting the sample.[23] The results in table 4 show that the coefficients of LIMIT are positive in all cases. This suggests that limit orders are more informative than market orders even after we control for order size, trading intensity, and prevailing spread and depth as well as return in the previous interval. This finding is consistent with the results of our nonparametric analysis. At the one-hour horizon, the LIMIT variable is significant at 1% for all deciles. Even at the one-day horizon it is still significant at 1% for almost all deciles. In addition, orders that are submitted at periods during which the trading intensity is high are typically more informative, both at the one-hour horizon and at the one-day horizon. As for the order size, while at the one-hour horizon there is a strong link between order size and order informativeness, at the one-day horizon the link seems to be weaker. Orders submitted when the spread is large are more likely to cause a quote change. Furthermore, a buy (sell) order submitted following a positive (negative) return is in general less (more) likely to induce the specialist to change the quote. Following a positive return, a new sell order is more likely to contain information different from the information that led to the positive return than a new buy order. The results are qualitatively similar if we use relative order size (i.e., order size divided by the quoted depth) instead of using absolute order size.

C. The Specialists' Perceptions

Given that the evidence presented thus far supports the hypothesis that limit orders are more informative, we next test whether specialists indeed perceive these orders to be more informative than market orders. To compare the *perceived informativeness*, we use the natural analogue of the test described in subection *B*. Specifically, we test which order type tends to induce specialists to increase (decrease) quotes following a buy (sell) order more often.

1. Inventory Considerations

Given that the focus of our model is the informativeness of the different order types, in the construction of the model we have decided to abstract from important inventory considerations that a specialist faces when trading with

23. The results for the split analysis are consistent with the ones presented and are available from the authors.

other participants.[24] In practice, however, there are two major possible reasons why a specialist changes the quotes after an order. First, the order might convey information about the true stock price, which we call the *information effect*. Second, the order might affect the specialist's inventory position, which we call the *inventory effect*.

An incoming market order has a direct effect on the specialist's inventory if the specialist trades with the order. A (nonmarketable) limit order, however, does not have a direct effect on a specialist's inventory position. Even when an order does not have a direct inventory effect, it may have an indirect effect. In particular, if the order is large enough to change considerably the limit order book, then it will have a significant indirect inventory effect in the sense that the ability and profitability of specialist inventory-motivated trading may be affected. Although a market order depletes the limit order book, a limit order inflates it. In summary, compared to an otherwise identical limit order, a market order usually has a different inventory impact.

Since our objective in this subsection is to measure whether the specialist perceives that limit orders contain more information than market orders, ideally we should control for the differential inventory impact of limit and market orders. To fully control for the inventory effect, one needs to have data on the specialist's inventory positions. One also needs to be able to identify the trades in which the specialist participates. Unfortunately, the TORQ data set does not contain information on a specialist's inventory position. Nor is it possible to clearly distinguish specialist trades from floor broker trades. Thus we are not able to completely control for the inventory effect. However, we are able to partially control for the direct inventory effect for a subsample of the orders. To do so we restrict ourselves to the subsample of orders that consists of only limit and market orders that are executed against the limit order book, since these orders do not have a direct inventory effect. If limit orders are more likely to induce specialists to change the prevailing quotes in the correct direction after controlling for the direct inventory effect, then we can conclude that the specialist perceives limit orders to convey more information than market orders if they have indirect inventory effects of similar magnitudes.

2. Quote Attribution Method

We first describe the general method in attributing quote price changes (i.e., changes in the midpoint of the spread) and then lay out the details.[25]

The general procedure is as follows:

- Limit order: If the next activity after a limit order is a quote change,

24. We are unaware of any comprehensive model that allows informed traders to use both market and limit orders and also takes into consideration inventory implications for the specialist.

25. The analysis for bid depth and ask depth changes yields similar qualitative results in terms of perceived informativeness and is omitted to save space.

we attribute the quote change to this limit order.

- Market order: In general, a market order can be either executed at the prevailing quotes, stopped by the specialist, or executed with an immediate price improvement. Thus in each one of the following cases we attribute a quote change to the given market order: (1) The market order is executed at the prevailing quote, and the execution is followed by a quote change; (2) the market order gets an immediate price improvement, in which case it could be that the quote change precedes the relevant transaction; and (3) the market order is stopped, and the next activity after the stop is a quote change.

In addition, for a quote change to be attributed to an order, the quote change must occur within one minute of the order submission.[26] It is easy to see that the above procedure never attributes a particular quote revision to both a market order and a limit order. Furthermore, some quote revisions are not attributed to either a market order or a limit order.

There are some details that need to be addressed when attributing quote revisions. First, if there are several market (limit) orders with the same time stamp, then a quote change will be attributed to at most one of the market (limit) orders.[27] Second, in very few cases a limit order and a market order have the same time stamp. In these cases we do not attribute the relevant quote change, if any, to either the limit order or the market order.[28] Third, the process of physically changing quotes takes several seconds. Thus the following scenario is possible: a specialist receives a limit order and decides to change the quotes. As she is in the process of changing the quotes, a limit order is reported via ITS. In such a case, the quote change may be mistakenly attributed to the limit order. Fortunately this problem arises only for extremely active stocks (the highest-activity decile). To partially adjust for this problem, we attribute a quote change to an order only if the event (any event, not only a limit [market] order) immediately preceding the order is at least 10 seconds away from the quote change.[29]

3. Comparing Perceived Informativeness

Given the above quote attribution method, we present the following definition.

Definition 3. Perceived informativeness of a given buy (sell) order type (i.e., either market or limit) is measured as the conditional probability that

26. For robustness we also used a quote attribution time ceiling of five minutes and an attribution rule without any time ceiling. In both cases the results are qualitatively similar to the ones reported in the paper.

27. This problem arises only for very actively traded stocks.

28. We also ran a version in which we attribute the relevant quote change to both the limit order and the market order. The results are similar.

29. In the whole sample, for less than 1% of the quote changes that appear after a market (limit) order, there is a limit (market) order that precedes the market (limit) order and is less than 10 seconds away from the quote change. For the highest-activity decile, the corresponding percentage is 3%.

the order induces the specialist to increase (decrease) the quote midpoint, as long as the attributed quote change occurs within one minute of submission of the buy (sell) order.

Restricting to the subsample of limit and market orders that are executed against the limit order book, table 5 shows the ratio of the conditional probability of the quote midpoint increasing (decreasing) after a limit buy (sell) order to that after a market buy (sell) order. It is the perceived informativeness counterpart of tables 2 and 3. Orders are partitioned into three "size" categories as in tables 2 and 3.

As can be seen from both panels A and B, for the high-activity deciles for small and medium orders, the conditional probability that the specialist increases (decreases) the quote midpoint is higher after a limit buy (sell) order than after a market buy (sell) order. Specifically, for these orders (both buy and sell), in almost all the cases the null of equal perceived informativeness is rejected at the 1% level against the alternative that limit orders are perceived as more informative. For the lower-activity deciles, the sample size is typically too small here to allow us to reliably compute the corresponding *p*-values for an individual activity decile. However, when we pool the bottom five activity deciles, consistent with the results for the high-activity deciles, we get for small and medium orders that limit orders are perceived more informative, at the 1% level.

For large order, panel A suggests that market buy orders are perceived as more informative than limit buy orders. However, on the sell side, the results suggest that their perceived informativeness is similar.[30] Panel B suggests that they are likely to be perceived as equally informative.

It is important to keep in mind that large market orders are more likely to result in an immediate price change due to the rules of trading because the specialist would let them "walk the book."[31] On the other hand, given that a trader who places a large market order is undertaking greater price risk, it could be that in fact these orders tend to be more informative than large limit orders. As our model demonstrates, if the mispricing is very large and the expected information horizon is short, an informed trader would optimally choose to submit a market order to guarantee execution. Thus it could be that these large market orders are placed by informed traders when the security is considerably mispriced and the expected information horizon is short.

As in subection *B*, to help discern potential patterns in behavior throughout the day, we have also performed the analysis separately for the beginning (9:30 a.m.–10:30 a.m.), the middle (10:30 a.m.–3:00 p.m.), and the end

30. Chan and Lakonishok (1993) documented the fact that institutional buy trades' permanent impact on prices is larger than the permanent impact of their sell trades. Saar (2001) develops a theoretical model to explain this asymmetry. However, he does not distinguish between market and limit orders. Keim and Madhavan (1995) show that some traders adopt different strategies on the buy and the sell sides.

31. We thank the referee for pointing this out.

TABLE 5 **Specialists' Perceptions**

Decile	Sell Orders			Buy Orders		
	Small	Medium	Large	Small	Medium	Large
	A. At the Quote					
Active	1.45	1.61	.90	1.50	1.32	.93
	{>.99}	{>.99}	{.06}	{>.99}	{>.99}	{.13}
	(.39; .27; 2,888)	(.37; .23; 353)	(.37; .41; 712)	(.39; .26; 2,927)	(.41; .31; 390)	(.41; .44; 794)
2	1.27	1.37	.95	1.17	1.27	.88
	{>.99}	{>.99}	{.30}	{>.99}	{.98}	{.09}
	(.31; .24; 539)	(.34; .25; 106)	(.35; .37; 302)	(.31; .27; 654)	(.32; .25; 106)	(.35; .40; 310)
3	1.11	1.35	.85	1.12	1.07	.81
	{.99}	{.99}	{.03}	{>.99}	{.71}	{.01}
	(.33; .30; 285)	(.31; .23; 79)	(.34; .40; 253)	(.35; .31; 307)	(.36; .33; 74)	(.36; .44; 223)
4	1.19	.99	.90	.80	1.03	.79
	{>.99}	{NA}	{.18}	{<.01}	{NA}	{.02}
	(.17; .15; 138)	(.24; .24; 38)	(.26; .29; 141)	(.17; .22; 189)	(.27; .27; 41)	(.27; .34; 146)
5	1.39	1.21	1.13	1.31	1.22	.84
	{>.99}	{NA}	{.85}	{>.99}	{NA}	{.07}
	(.28; .20; 96)	(.29; .24; 35)	(.32; .28; 153)	(.30; .23; 117)	(.30; .24; 45)	(.31; .37; 177)
6	1.19	1.02	1.00	1.34	1.33	.79
	{.97}	{NA}	{.50}	{>.99}	{NA}	{.06}
	(.23; .19; 57)	(.25; .25; 24)	(.31; .31; 110)	(.22; .17; 61)	(.27; .20; 24)	(.28; .35; 104)
7	1.81	1.68	.88	1.40	1.46	.77
	{NA}	{NA}	{.23}	{NA}	{NA}	{.06}
	(.19; .10; 29)	(.24; .14; 15)	(.27; .30; 77)	(.23; .16; 47)	(.26; .18; 20)	(.26; .34; 83)
8	1.73	1.03	.83	1.31	1.05	.93
	{NA}	{NA}	{.14}	{NA}	{NA}	{.38}
	(.21; .12; 21)	(.25; .24; 14)	(.28; .34; 66)	(.24; .19; 28)	(.27; .25; 14)	(.27; .29; 64)

TABLE 5 ***(Continued)***

Decile	Sell Orders			Buy Orders		
	Small	Medium	Large	Small	Medium	Large
9	1.46	1.59	.64	2.06	2.53	.85
	{NA}	{NA}	{NA}	{NA}	{NA}	{.24}
	(.17; .12; 11)	(.26; .16; 7)	(.21; .32; 36)	(.24; .12; 12)	(.24; .09; 7)	(.28; .33; 51)
Inactive	1.31	1.09	1.00	1.18	·	.88
	{NA}	{NA}	{NA}	{NA}	{NA}	{NA}
	(.16; .12; 4)	(.27; .25; 2)	(.30; .30; 18)	(.23; .19; 4)	(.13; .00; 1)	(.27; .31; 22)
	B. Better than the Quote					
Active	1.31	1.61	.89	1.40	1.38	.79
	{>.99}	{>.99}	{.11}	{>.99}	{>.99}	{.01}
	(.42; .32; 1,900)	(.40; .25; 243)	(.33; .38; 423)	(.43; .31; 1,927)	(.43; .31; 244)	(.38; .48; 484)
2	1.41	1.87	.94	1.27	2.17	.72
	{>.99}	{>.99}	{.37}	{>.99}	{>.99}	{.02}
	(.38; .27; 386)	(.37; .20; 80)	(.32; .34; 118)	(.37; .30; 394)	(.34; .16; 67)	(.36; .49; 121)
3	1.52	2.63	1.18	1.59	1.71	1.21
	{>.99}	{>.99}	{.87}	{>.99}	{>.99}	{.91}
	(.39; .26; 241)	(.41; .15; 70)	(.35; .30; 102)	(.42; .26; 216)	(.44; .26; 62)	(.38; .31; 89)
4	1.44	2.40	1.05	1.52	2.16	1.00
	{>.99}	{NA}	{.60}	{>.99}	{NA}	{.49}
	(.31; .22; 99)	(.39; .16; 33)	(.35; .33; 57)	(.37; .24; 122)	(.41; .19; 39)	(.35; .36; 57)
5	1.88	2.69	1.27	1.51	3.11	1.35
	{>.99}	{NA}	{NA}	{>.99}	{NA}	{.80}
	(.39; .21; 78)	(.37; .14; 28)	(.34; .27; 45)	(.40; .27; 103)	(.43; .14; 44)	(.36; .27; 62)
6	2.16	2.90	1.22	1.76	3.51	2.07
	{NA}	{NA}	{NA}	{>.99}	{NA}	{NA}
	(.43; .20; 46)	(.38; .13; 18)	(.38; .31; 38)	(.36; .21; 61)	(.36; .10; 24)	(.37; .18; 47)
7	2.35	1.55	1.71	1.74	1.19	1.27
	{NA}	{NA}	{NA}	{NA}	{NA}	{NA}
	(.36; .15; 31)	(.39; .25; 15)	(.34; .20; 26)	(.42; .24; 46)	(.34; .29; 16)	(.35; .28; 34)

8	2.26	·	1.39	4.21	2.26	.46
	{NA}	{NA}	{NA}	{NA}	{NA}	{NA}
	(.39; .17; 20)	(.49; .00; 13)	(.32; .23; 20)	(.41; .10; 27)	(.38; .17; 13)	(.30; .64; 21)
9	3.92	2.64	1.44	2.58	·	1.22
	{NA}	{NA}	{NA}	{NA}	{NA}	{NA}
	(.40; .10; 9)	(.29; .11; 4)	(.48; .33; 19)	(.41; .16; 15)	(.54; .00; 9)	(.47; .38; 24)
Inactive	4.41	·	2.38	·	·	·
	{NA}	{NA}	{NA}	{NA}	{NA}	{NA}
	(.29; .07; 2)	(.29; .00; 1)	(.40; .17; 6)	(.28; .00; 3)	(.37; ·; 2)	(.42; .00; 8)

NOTE.—This table presents the ratio of the conditional probability of the quote midpoint changing in the *correct direction* (up after a buy, down after a sell) given a submission of a limit order to the conditional probability given a submission of a market order, while controlling for order size relative to the quoted depth. Only market orders that are executed against the book are considered. See also the note to table 2.

(3:00 p.m.–4:00 p.m.) of the trading day.[32] For small and medium orders, for all three time intervals, the results are similar to the ones reported above. For large orders, in the beginning and the middle of the trading day the specialist perceives market orders as more informative, similar to panel A of table 5. However, for the end of the trading day the evidence seems to suggest that large limit orders actually induce specialists to revise their quotes significantly more often than large market orders.[33]

To shed some light on the above results, one should differentiate between the information set of the specialist and the information set of other market participants. Except for floor brokers, who may have access to the limit order book at the discretion of the specialist, other market participants do not observe it. In recent years the willingness of specialists on the NYSE to display the limit order book to floor brokers has increased considerably, thereby increasing the transparency of the market. In the early 1990s, specialists were much less forthcoming. Thus, even if a limit order conveyed more information to the specialist, it was hard for other market participants, including those on the floor, to evaluate whether the specialist changed the quotes as a result of new information or for inventory control.

Given NYSE Rule 79A.15 (which specifies that the specialist must display a limit order within 30 seconds), one might hastily conclude that our conclusion, which states that apart from large orders in the morning and the middle of the day the specialist perceives limit orders as more informative, is a result of the NYSE order-processing rules. In fact, Rule 79A.15 was adopted only in October 1997, well after our data sample period of 1990:11–1991:1.[34] At the end of 1990, there were very vague guidelines about when a specialist was obligated to display a limit order. As pointed out by McInish and Wood (1995), the procedure of displaying limit orders was far from automatic at the time, and specialists had considerable discretion in deciding when to display hidden orders. As a result, the limit order book contained a significant number of hidden orders, and there was no formal obligation to represent the best limit order on the book as the prevailing quote.[35] Interested readers are referred to Appendix B, where we give a brief description of the evolution of limit order–processing rules on the NYSE in the past decade.

32. Results are available from the authors.

33. The results do not change if we exclude program trades.

34. It may well be true that after the adoption of NYSE Rule 79A.15, our approach of using the quote revision as a measure of perceived order informativeness no longer applies. However, our main qualitative results regarding order submission strategy and order informativeness should still hold as long as, on average, the information horizon is not too short, even though some other measures such as depth revisions beyond the submitted limit orders can become more appropriate.

35. While a specialist is also supposed to change the quote to reflect the presence of a stop, specialists probably did not do this very reliably during the period covered by the TORQ database either. In any case, such an automatic procedure biases against finding that the specialist perceives limit orders to be more informative.

IV. Conclusions

We have demonstrated that allowing informed traders to decide optimally on whether to submit limit or market orders can generate equilibria in which not only informed traders prefer placing limit orders but also limit orders are more informative than market orders. Our empirical results support the hypothesis that limit orders are more informative. This is in contrast to the standard modeling assumption that informed traders prefer placing market orders, but consistent with the experimental findings of Bloomfield et al. (2003). It implies in particular that private information about future security returns may not be short lived and the fraction of informed traders may be significant.

As could be expected, the trade-off between execution risk and price uncertainty determines the order type selected. Two important factors that affect this trade-off are the horizon of the private information and the magnitude of mispricing. First, longer-lived information decreases the execution risk of placing a limit order, thereby increasing the probability that an informed trader submits a limit order. This shift in order placement strategy by informed traders also leads to a decrease in the equilibrium bid-ask spread. Second, greater magnitude of mispricing increases the implicit cost of nonexecution, tilting informed traders toward market orders. Interestingly, in equilibrium, even when the asset value lies outside the quoted spread, as long as the mispricing is not severe, an informed trader may still prefer to place limit orders.

In addition, in contrast to the literature, we show that as the proportion of uninformed traders increases, an informed trader is less likely to place a limit order even though the profitability of a limit order becomes greater. The reason is that the profitability of a market order increases even more than that of a limit order as the proportion of uninformed traders increases.

Appendix A

Throughout this appendix we shall use the following definitions:

$$h_1(a_1,\ b_2) \equiv \frac{\dfrac{\mu(1-l)}{2(1-\mu)}m + \int_{\max(c,a_1)}^{\bar{v}} xg(x)dx}{\dfrac{\mu(1-l)}{2(1-\mu)} + \int_{\max(c,a_1)}^{\bar{v}} g(x)dx}$$

and

$$h_2(a_1,\ b_2) \equiv \frac{\dfrac{\mu l}{2(1-\mu)}m + \int_{\max(m,b_2)}^{c} xg(x)dx + l\int_{\underline{v}}^{b_2} xg(x)dx}{\dfrac{\mu l}{2(1-\mu)} + \int_{\max(m,b_2)}^{c} g(x)dx + l\int_{\underline{v}}^{b_2} g(x)dx},$$

where unless specified otherwise $b_2 \equiv b_2(LB)$ throughout. Furthermore, let

$$f_1(a_1, b_2, \mu, l, p) = \frac{\mu}{2(1-\mu)}(1-l)(m-a_1) + \int_c^{\bar{v}} (x-a_1)g(x)dx,$$

$$f_2(a_1, b_2, \mu, l, p) = \frac{\mu l}{2(1-\mu)}(m-b_2) + \left[\int_{\max(m,b_2)}^{c} (x-b_2)g(x)dx + l\int_v^{b_2} (x-b_2)g(x)dx\right],$$

where

$$c \equiv \frac{a_1 - \mu(pb_2/2)}{1-\mu(p/2)}.$$

So in equilibrium

$$f_1(a_1, b_2, \mu, l, p) = 0,$$

$$f_2(a_1, b_2, \mu, l, p) = 0.$$

LEMMA 3.

$$\frac{da_1}{dp} = -\frac{1}{H}\frac{\partial f_1}{\partial c}\frac{\partial c}{\partial p}\frac{\partial f_2}{\partial b_2},$$

$$\frac{da_1}{dl} = -\frac{1}{H}\left[\left(\frac{\partial f_2}{\partial c}\frac{\partial c}{\partial b_2} + \frac{\partial f_2}{\partial b_2}\right)\frac{\partial f_1}{\partial l} - \frac{\partial f_1}{\partial c}\frac{\partial c}{\partial b_2}\frac{\partial f_2}{\partial l}\right],$$

$$\frac{db_2}{dp} = -\frac{1}{H}\frac{\partial f_2}{\partial c}\frac{\partial f_1}{\partial a_1}\frac{\partial c}{\partial p},$$

$$\frac{db_2}{dl} = -\frac{1}{H}\left[-\frac{\partial f_2}{\partial c}\frac{\partial c}{\partial a_1}\frac{\partial f_1}{\partial l} + \left(\frac{\partial f_1}{\partial c}\frac{\partial c}{\partial a_1} + \frac{\partial f_1}{\partial a_1}\right)\frac{\partial f_2}{\partial l}\right],$$

$$\frac{dc}{dp} = \frac{1}{H}\frac{\partial c}{\partial p}\frac{\partial f_1}{\partial a_1}\frac{\partial f_2}{\partial b_2},$$

$$\frac{dc}{dl} = -\frac{1}{H}\left(\frac{\partial c}{\partial a_1}\frac{\partial f_2}{\partial b_2}\frac{\partial f_1}{\partial l} + \frac{\partial c}{\partial b_2}\frac{\partial f_1}{\partial a_1}\frac{\partial f_2}{\partial l}\right),$$

where

$$H = \frac{\partial f_1}{\partial c}\frac{\partial c}{\partial a_1}\frac{\partial f_2}{\partial b_2} + \frac{\partial f_2}{\partial c}\frac{\partial c}{\partial b_2}\frac{\partial f_1}{\partial a_1} + \frac{\partial f_1}{\partial a_1}\frac{\partial f_2}{\partial b_2} > 0.$$

Proof. The expressions for da_1/dp, db_2/dp, da_1/dl, and db_2/dl[36] are obtained by using the implicit function theorem.

36. In computations of these derivatives, via the implicit function theorem, for the first (second) pair, μ and l (p) are treated as constants so that f_i becomes $f_i(a_1, b_2, p)$ ($f_i(a_1, b_2, l)$).

Taking a total derivative of c with respect to p and l, we obtain

$$\frac{dc}{dp} = \frac{\partial c}{\partial p} + \frac{\partial c}{\partial a_1}\frac{da_1}{dp} + \frac{\partial c}{\partial b_2}\frac{db_2}{dp},$$

$$\frac{dc}{dl} = \frac{\partial c}{\partial l} + \frac{\partial c}{\partial a_1}\frac{da_1}{dl} + \frac{\partial c}{\partial b_2}\frac{db_2}{dl}.$$

Plugging in the expressions for da_1/dp, db_2/dp, da_1/dl, and db_2/dl, followed by some tedious algebra, the above simplifies to the provided expressions for dc/dp and dc/dl. QED

Proof of Proposition 1

First, note that the fact that the market maker's updating rule is a deterministic function of the limit price rules out mixed strategies on the part of either the informed or the patient uninformed traders.

A limit order can get placed only at time 1 and can execute only against an incoming market order. Thus it is evident that given a quote-updated rule $b_2(LB;\ PB)$, any individual placing a limit buy will place it at the minimum price PB that satisfies $b_2(LB;\ PB^*) \leq PB^*$. By continuity, in PB of $b_2(LB;\ PB)$, we obtain that $b_2(LB;\ PB^*) = PB^*$.

A symmetric argument holds for the sell side. QED

Proof of Proposition 2

The proof is obtained by proving three lemmas. The first shows that there exists a solution to (5) and (6). The second shows that the bid after a submission of a limit order is above the original bid. The third shows that the solution that is found in the first part satisfies the participation constraint (7).

LEMMA 4. There exists (a_1, b_2) that solves (5) and (6).

Proof. It is straightforward to show that

$$m^- < h_1(a_1,\ b_2) < m^+ + \frac{2(1-\mu)}{\mu(1-l)}\int_0^{\bar{v}} xg(x)dx < \infty$$

and

$$-\infty < m^- - \frac{2(1-\mu)}{\mu}\int_0^{\bar{v}} xg(x)dx < h_2(a_1,\ b_2) < m^+ + \frac{2(1-\mu)}{\mu l}\int_0^{\bar{v}} xg(x)dx < \infty,$$

where $m^- = \min(m, 0)$ and $m^+ = \max(m, 0)$. Let

$$\overline{h} = m^+ + \max\left[\frac{2(1-\mu)}{\mu(1-l)},\ \frac{2(1-\mu)}{\mu l}\right]\int_0^{\bar{v}} xg(x)dx$$

and

$$\underline{h} = m^- - \frac{2(1-\mu)}{\mu}\int_0^{\bar{v}} xg(x)dx.$$

Since h_1 and h_2 are both continuous functions, by Brouwer's fixed-point theorem, there exists $(a_1^*, b_2^*) \in (\underline{h}, \overline{h})$, which solves $a_1^* = h_1(a_1^*, b_2^*)$ and $a_2^* = h_2(a_1^*, b_2^*)$; that is, (a_1^*, b_2^*) solves (5) and (6). QED

LEMMA 5. $b_2 > b_1$.

Proof. Since $m \geq b_1$, it holds trivially for the case $b_2 > m$. For the rest of the proof we assume $m > b_2$.

Using equation (5), we obtain

$$a_1 - m = \frac{\int_c^{\bar{v}} (x-m) g(x) dx}{[\mu/2(1-\mu)](1-l) + \int_c^{\bar{v}} g(x) dx},$$

and from equation (6),

$$b_2 - m = \frac{\int_{\max(m,b_2)}^{c} (x-m)g(x)dx + l\int_{\underline{v}}^{b_2}(x-m)g(x)dx}{[\mu/2(1-\mu)]l + \int_{\max(m,b_2)}^{c} g(x)dx + l\int_{\underline{v}}^{b_2} g(x)dx}$$
$$> \frac{\int_m^c (x-m)g(x)dx + l\int_{\underline{v}}^{b_2}(x-m)g(x)dx}{[\mu/2(1-\mu)]l + l\int_c^{\bar{v}} g(x)dx},$$

where the inequality follows from combining the fact that $m > b_2$ and $b_2 > -c$. Since g is symmetric around its mean, it suffices to show that $b_2 - m > -(a_1 - m)$, which will hold if

$$\left[\int_m^c (x-m)g(x)dx + l\int_{\underline{v}}^{b_2}(x-m)g(x)dx\right]\left[\frac{\mu}{2(1-\mu)}(1-l) + \int_c^{\bar{v}} g(x)dx\right] >$$
$$-\left[\int_c^{\bar{v}}(x-m)g(x)dx\right]\left[\frac{\mu}{2(1-\mu)}l + l\int_{\underline{v}}^{b_2} g(x)dx\right]$$

or equivalently if

$$\left[\int_c^{\bar{v}} g(x)dx\right]\left[\int_m^c (x-m)g(x)dx + l\int_{\underline{v}}^{b_2}(x-m)g(x)dx + l\int_c^{\bar{v}}(x-m)g(x)dx\right]$$
$$+ \frac{\mu}{2(1-\mu)}\left[l\int_c^{\bar{v}}(x-m)g(x)dx + (1-l)\int_m^c (x-m)g(x)dx\right]$$
$$+ \frac{\mu}{2(1-\mu)} l \int_{\underline{v}}^{b_2}(x-m)g(x)dx > 0,$$

which holds since $m > b_2$. QED

LEMMA 6. If (a_1, b_2) solves (5) and (6), then the participation constraint (7) is satisfied.

Proof. Using (5) to plug in for $\frac{1}{2}\mu m$ in (7), we obtain that it suffices to show that

$$(1-p)m + p\{\mu\tfrac{1}{2}[b_2 + lm + (1-l)a_1]\}$$
$$+ (1-\mu)p\left[\int_{\underline{v}}^{b_2} b_2 g(x)dx + \int_{b_2}^{c} xg(x)dx + \int_c^{\bar{v}} a_1 g(x)dx\right] < a_1.$$

By lemma 1, $a_1 > \max(m, b_2)$, which implies, after some basic algebra, that it suffices to show that

$$\int_{a_1}^{c} (x - a_1) g(x) dx - \int_{\underline{v}}^{b_2} (a_1 - b_2) g(x) dx < 0.$$

By the definition of c, one obtains that $0 < c - a_1 < a_1 - b_2$. Combining this fact with the symmetry of $g(v)$ around its mean m implies that it is sufficient to show that

$$\int_{a_1}^{c} g(x) dx - \int_{\underline{v}}^{b_2} g(x) dx < 0,$$

or, equivalently,

$$\int_{m-(c-m)}^{m-(a_1-m)} g(x) dx - \int_{\underline{v}}^{b_2} g(x) dx < 0,$$

which follows from the fact that $\underline{v} < m - (c - m)$ and $m - (a_1 - m) = b_1 < b_2$. QED

Proof of Lemma 1

From (5), if $\max(c, a_1) \geq \bar{v}$, then $a_1 = m$, which contradicts the fact that $\bar{v} > m$. As a result, the second term in (5) is always positive, implying that $a_1 > m$.

If $a_1 \leq b_2$, then $c \leq b_2$, so that (6) reduces to

$$\frac{\mu l}{2(1 - \mu)} (m - b_2) + l \int_{\underline{v}}^{b_2} (x - b_2) g(x) dx = 0.$$

Since the second term on the left-hand side is nonpositive, $b_2 \leq m$. This contradicts the assumption that $a_1 \leq b_2$, since we have shown above that $a_1 > m$. Therefore, $a_1 > b_2$, and by the definition of c, this also implies that $c > a_1$.

Finally, to show that $b_2 > \underline{v}$, note that if $b_2 \leq \underline{v}$, then the last term on the left-hand side of (6) is zero, and both the first and the second terms are strictly positive, therefore a contradiction. QED

Proof of Theorem 1

Since $g(v)$ is symmetric around its mean m and the uninformed trader is equally likely to place a buy or a sell, the ex ante probability of an informed trader using a limit order at date 1 is

$$\begin{aligned}
&\Pr(\text{limit order submitted} | \text{trader informed}) \\
&= \Pr(LB | \text{trader informed}) + \Pr(LS | \text{trader informed}) \\
&= 2 \Pr(LB | \text{trader informed}) \\
&= 2 \int_{\max(m, b_2)}^{c} g(x) dx.
\end{aligned}$$

Without loss of generality we prove the theorem for the case $m = 0$.

It is straightforward to show that there exists $\mu^*(l, p) \in (0, 1)$ such that

$$\left[1 - \frac{p\mu^*(l, p)}{2}\right]c = h_1\left(\left[1 - \frac{p\mu^*(l, p)}{2}\right]c,\ 0\right),$$
$$0 = h_2\left(\left[1 - \frac{p\mu^*(l, p)}{2}\right]c,\ 0\right),$$

where c is the solution to

$$\int_0^c xg(x)dx = l\int_0^{\bar{v}} xg(x)dx > 0.$$

Since $g(v)$ is symmetric around zero,

$$\frac{\int_0^c xg(x)dx}{\int_0^c g(x)dx} < 2\int_0^{\bar{v}} xg(x)dx,$$

which together with $\int_0^c xg(x)dx = l\int_0^{\bar{v}} xg(x)dx$ implies $\int_0^c g(x)dx > l/2$.

For the second part of the theorem, the fact that $m = b_2 = 0$ implies that

$$\Pr(\text{order is by informed}|\text{limit order submitted}) >$$
$$\Pr(\text{order is by informed}|\text{market order submitted})$$

is equivalent to

$$\frac{(1-\mu)\Pr(\text{limit submitted}|\text{trader informed})}{\mu l + (1-\mu)\Pr(\text{limit submitted}|\text{trader informed})} >$$
$$\frac{(1-\mu)\Pr(\text{market submitted}|\text{trader informed})}{\mu(1-l) + (1-\mu)\Pr(\text{market submitted}|\text{trader informed})},$$

where Pr(market submitted | trader informed) = 1 − Pr(limit submitted | trader informed). Some simple algebra yields that this condition is equivalent to

$$\frac{\Pr(\text{limit submitted}|\text{trader informed})}{1-\Pr(\text{limit submitted}|\text{trader informed})} > \frac{l}{1-l},$$

and combining the fact that for $x < 1$ $x/(1-x)$ increases in x with the fact that for $(l, p, \mu^*(l, p))$ $\Pr(\text{limit submitted}|\text{trader informed}) > l$ yields the proof. QED

Proof of Lemma 2

Since $m > b_2$, the derivatives with respect to p and l of Pr(limit | informed) are proportional to dc/dp and dc/dl, respectively. The expressions for dc/dp and dc/dl are provided in lemma 3.

Equation (6) implies that $df_2/dl < 0$. All the other partial derivatives that appear in the expressions for dc/dp and dc/dl are easily signed, and the results follow. QED

Proof of Proposition 3

We first present two auxiliary lemmas.

LEMMA 7. For each density g there exists an $l^* < 1$ such that $l > l^* \Rightarrow m > b_2(LB)$.

Proof. Assume by contradiction that $b_2 > m$ throughout. This implies that the expression in braces in equation (6) is bounded above by

$$\int_m^c (x-m)g(x)dx + l\int_{\underline{v}}^m (x-m)g(x)dx.$$

Given the symmetry of g around its mean,

$$\int_m^c (x-m)g(x)dx + \int_{\underline{v}}^m (x-m)g(x)dx < 0.$$

Thus, for $l < 1$ large enough, the term in braces in equation (6) is negative, implying that $m > b_2$, a contradiction. QED

LEMMA 8. If $b_2 > m$, then $da_1/d\mu < 0$.

Proof.

$$\frac{da_1}{d\mu} = -\frac{1}{H}\left(\frac{\partial f_2}{\partial b_2}\frac{\partial f_1}{\partial c}\frac{\partial c}{\partial \mu} + \frac{\partial f_2}{\partial b_2}\frac{\partial f_1}{\partial \mu} - \frac{\partial f_1}{\partial c}\frac{\partial c}{\partial b_2}\frac{\partial f_2}{\partial \mu}\right),$$

where H is defined in lemma 3. QED

The first part of the proposition follows immediately from combining lemma 7 with lemma 2.

Given lemma 2, we can restrict the proofs to the case $b_2 > m$, in which case

$$\begin{aligned}\frac{d\Pr(\text{limit}|\text{informed})}{dp} &= g(c)\frac{dc}{dp} - g(b_2)\frac{db_2}{dp}\\ &= g(c)\frac{1}{H}\frac{\partial c}{\partial p}\frac{\partial f_1}{\partial a_1}\frac{\partial f_2}{\partial b_2} - g(b_2)\frac{-1}{H}\frac{\partial f_2}{\partial c}\frac{\partial f_1}{\partial a_1}\frac{\partial c}{\partial p},\end{aligned}$$

$$\begin{aligned}\frac{d\Pr(\text{limit}|\text{informed})}{dl} &= \left[g(c)\frac{dc}{dl} - g(b_2)\frac{db_2}{dl}\right]\\ &= g(c)\frac{-1}{H}\left(\frac{\partial c}{\partial a_1}\frac{\partial f_2}{\partial b_2}\frac{\partial f_1}{\partial l} + \frac{\partial c}{\partial b_2}\frac{\partial f_1}{\partial a_1}\frac{\partial f_2}{\partial l}\right)\\ &\quad - g(b_2)\frac{-1}{H}\left(\frac{\partial c}{\partial a_1}\frac{\partial f_1}{\partial c}\frac{\partial f_2}{\partial l} + \frac{\partial f_1}{\partial a_1}\frac{\partial f_2}{\partial l} - \frac{\partial c}{\partial a_1}\frac{\partial f_2}{\partial c}\frac{\partial f_1}{\partial l}\right),\end{aligned}$$

where H is defined in lemma 3.

Using the fact that $b_2 > m$ and simplifying yields that $d\Pr(\text{limit}|\text{informed})/dp$ has the same sign as

$$\left[\frac{\mu l}{2(1-\mu)} + l\int_{\underline{v}}^{b_2} g(x)dx + \int_{b_2}^{c} g(x)\right] - g(b_2)(c - b_2) \qquad \text{(A1)}$$

and $d\Pr(\text{limit}|\text{informed})/dl$ has the same sign as

$$g(c)\Bigg\{\bigg[\frac{\mu l}{2(1-\mu)} + l\int_{\underline{v}}^{b_2} g(x)dx + \int_{b_2}^{c} g(x)\bigg](a_1 - m)$$
$$+\bigg[\frac{p\mu}{2}(1-l) + \frac{g(b_2)}{g(c)}\bigg(1-\frac{p\mu}{2}\bigg)(1-l) + g(b_2)\frac{(1-\mu)p}{4}(c-b_2)\bigg]$$
$$\cdot\bigg[\frac{\mu}{2(1-\mu)}(b_2-m) + \int_{\underline{v}}^{b_2}(b_2-x)g(x)dx\bigg] - g(b_2)(c-b_2)(a_1-m)\Bigg\}. \quad \text{(A2)}$$

For the second part of the proposition, $b_2 > m$ implies

$$c - b_2 = \frac{a_1 - b_2}{1-(p\mu/2)} < 2(a_1 - m).$$

Combining this with lemma 4 implies that there exists a $\mu^* < 1$ such that $\mu > \mu^*$ implies $\mu l/2(1-\mu) > g(b_2)(c-b_2)$, which shows that for $\mu > \mu^*$ both equations (A1) and (A2) are positive.

For the third part,

$$b_2 - m < a_1 - m < \frac{2(1-\mu)}{\mu}\frac{1}{1-l}\int_0^{\bar{v}} xg(x)dx,$$

where the second inequality follows from equation (5).

Equation (6) can be written as

$$\int_{b_2}^{c}(x-b_2)g(x)dx = l\bigg[\frac{\mu l}{2(1-\mu)}(b_2-m) + \int_{\underline{v}}^{b_2}(b_2-x)g(x)dx\bigg].$$

Therefore, the fact that

$$|b_2| + \int_{\underline{v}}^{\bar{v}}|x|g(x)dx > \int_{\underline{v}}^{b_2}(b_2-x)g(x)dx > \int_{\underline{v}}^{m}(m-x)g(x)dx > 0$$

implies that as l goes to zero b converges to c. Thus there exists a $0 < \tilde{l}$ such that $l < \tilde{l}$ implies that equation (A2) is positive. QED

For the last part, using lemma 3 in conjunction with the facts that g is a uniform distribution and $b_2 > m$ yields that $g(c)(dc/dp) - g(b_2)(db_2/dp)$ has the same sign as $-[(\partial f_2/\partial b_2) + (\partial f_2/\partial c)]$.

Furthermore, after some algebra, it can be shown that $g(c)(dc/dp) - g(b_2)(db_2/dp)$ has the same sign as

$$-\frac{\partial c}{\partial a_1}\frac{\partial f_1}{\partial l}\bigg(\frac{\partial f_2}{\partial b_2} + \frac{\partial f_2}{\partial c}\bigg) + \frac{\partial f_1}{\partial c}\frac{\partial f_2}{\partial l} - \frac{\partial f_2}{\partial l}\frac{\partial f_1}{\partial a_1}\bigg(\frac{\partial c}{\partial b_2} - 1\bigg),$$

which is positive if $-[(\partial f_2/\partial b_2) + (\partial f_2/\partial c)]$ is positive:

$$-\bigg(\frac{\partial f_2}{\partial b_2} + \frac{\partial f_2}{\partial c}\bigg) = \frac{\mu l}{2(1-\mu)} + l\int_{\underline{v}}^{b_2} g(x)dx + \int_{b_2}^{c} g(x)dx - g(b_2)(c-b_2),$$

which is positive when g is uniform. QED

Proof of Proposition 4

The result for da_1/dp follows directly from lemma 3. Furthermore, lemma 3 implies that

$$\frac{da_1}{dl} \propto -\frac{\partial c}{\partial b_2}\left(\frac{\partial f_2}{\partial c}\frac{\partial f_1}{\partial l} - \frac{\partial f_1}{\partial c}\frac{\partial f_2}{\partial l}\right) - \frac{\partial f_2}{\partial b_2}\frac{\partial f_1}{\partial l},$$

which is positive if the term in the parentheses is positive. The term inside the parentheses takes the form

$$g(c)\left\{\frac{\mu}{2(1-\mu)}[(c-a)(m-b_2)+(c-b_2)(a-m)] - (c-a)\int_{\underline{x}}^{b_2}(b_2-x)g(x)dx\right\}.$$

Furthermore, after some simple manipulations, equation (7) can be written as

$$\int_{\underline{x}}^{b_2}(b_2-x)g(x)dx < \frac{a-(p\mu/2)b_2}{p(1-\mu)} - m\frac{1-(p\mu/2)}{p(1-\mu)}.$$

Thus it suffices to show that

$$\frac{\mu}{2(1-\mu)}[(c-a)(m-b_2)+(c-b_2)(a-m)]$$
$$-(c-a)\left[\frac{a-(p\mu/2)b_2}{p(1-\mu)} - m\frac{1-(p\mu/2)}{p(1-\mu)}\right] \geq 0.$$

Simplifying this equation yields that the left-hand side is in fact identically zero. QED

Appendix B

Evolution of Limit Order–Processing Rules

A person who is not aware of the order-processing rules that were in place on the NYSE at the end of 1990 might hastily conclude that our conclusion that limit orders convey more information is an artifact of the order-processing rules and procedures on the NYSE. Rule 79A.15 states that upon receipt of a customer limit order, a specialist must display the order within 30 seconds. Specifically, the specialist must publish a bid or an offer that reflects the price and full size of the order if it is at a price that would improve the bid or offer and the full size of the order if it is priced at the same price as the current bid or offer, and the current bid or offer is equal to the national best bid or offer. The specialist would not, however, be required to add size to the prevailing bid or offer if the size of the customer's limit order represents a de minimis increase (i.e., 10% or less) over the size of the prevailing bid or offer.[37]

While Rule 79A.15 does not seem to leave much leeway for the specialist in deciding whether to update the quote following a submission of a limit order, things were quite different at the end of 1990. McInish and Wood (1995), using the TORQ data set, demonstrated that a substantial portion of limit orders were in fact hidden. Their paper

37. For additional exemptions, see details of Rule 79A.15.

received considerable attention after a *Wall Street Journal* article, appearing in late 1991, discussed their results. At the time, there were very vague guidelines as to when a specialist is obligated to display a limit order that was received through the SuperDot system. As they pointed out, the procedure of displaying limit orders was far from automatic. A detailed account of their results appeared later in McInish and Wood (1995). In an NYSE working paper that was written as part of the documentation for the TORQ data set, Hasbrouck and Sosebee (1992, 10) reported that "the specialist is under no obligation to represent the limit order as the prevailing bid if he does not feel that the new price is representative of the market in that issue." It is important to bear in mind that the sizes of the quotes are also representative. When a quote is the specialist's, he is under no obligation to represent the full extent of his interest at that price. When the quote comes from the Display Book, the specialist has no obligation to represent the full order on the book; he could post a smaller amount.

Only in March 1993 did the NYSE issue Information Memo 93-12 ("Exposure of SuperDot Limit Orders at Their Limit Prices"), which specified that a limit order received through SuperDot will be assumed to be requested to be displayed and that the specialist should display the price of a limit order that implies a price improvement within two minutes. Even after memo 93-12 was issued, some traders still kept accusing specialists of holding back limit orders (See *Investment Dealers' Digest* 8 [February 14, 1994]). In October 1995, the NYSE issued Information Memo 95-39 ("Showing Full Size of SuperDot Electronic Book Orders"), which extended the requirement to display the size of the orders.

In September 1996 the Securities and Exchange Commission adopted Rule 11Ac1-4 ("Display Rule"), which requires a specialist to display the price and the full size of customer limit orders that would improve the bid or offer in a security no later than 30 seconds after receiving them. Subsequently in October 1997 the NYSE adopted Rule 79A.15.

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