The Capital Asset Pricing Model

Capital Budgeting and Corporate Objectives

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Overview

- Utility and risk aversion
  - Choosing efficient portfolios
- Investing with a risk-free asset
  - Borrowing and lending
  - The market portfolio
  - The Capital Market Line (CML)
- The Capital Asset Pricing Model (CAPM)
  - The Security Market Line (SML)
  - Beta
  - Project analysis
Efficient Portfolios with Multiple Assets

Investors prefer the Efficient Frontier, which includes portfolios of Asset 1 and Asset 2 among others.

Introducing a Riskfree Asset

- Suppose we introduce the opportunity to invest in a riskfree asset.
  - How does this alter investors’ portfolio choices?

- The riskfree asset has a zero variance, and zero covariance with every other asset (or portfolio).
  - $\text{var}(r_f) = 0$.
  - $\text{cov}(r_f, r_j) = 0$ for all $j$.

- What is the expected return and variance of a portfolio consisting of a fraction $(1-\alpha)$ of the riskfree asset and a fraction $\alpha$ of the risky asset (or portfolio of risky assets)?
Risk and Return with a Riskfree asset

- Expected Return
  \[ E[r_p] = \alpha E[r_j] + (1 - \alpha) r_f \]

- Variance and Standard Deviation
  \[ Var[r_p] = \sigma_p^2 = \alpha^2 \sigma_j^2 \Rightarrow \sigma_p = \alpha \sigma_j \]

- Hence, the risk-return tradeoff is:
  \[ E[r_p] = r_f + \frac{(E[r_j]-r_f)}{\sigma_j} \sigma_p \]

  » The slope is called the Sharpe-Ratio

Risk and Return with a Riskfree asset

![Graph](image)

- The line represents all portfolios depending on \( \alpha \)
Investing with Borrowing and Lending

The Capital Market Line I
The Capital Market Line II

- The CML gives the tradeoff between risk and return for portfolios consisting of the riskfree asset and the tangency portfolio M. The equation of the CML is:

\[ E(r_p) = r_f + \sigma_p \frac{E(r_M) - r_f}{\sigma_M} \]

- The expected rate of return on a risky asset can be thought of as composed of two terms.
  - The return on a riskfree security, like U.S. Treasury bills; compensating investors for the time value of money.
  - A risk premium to compensate investors for bearing risk.

\[ E(r_p) = r_t + \text{Risk x [Market Price of Risk]} \]
Decomposition of Return

What is the “M” Portfolio?

- Everybody holds the tangency portfolio M
  - If all hold the same portfolio, it must be the market!
- Nobody can do better than holding the market
- Write the weight of asset $j$ in the market portfolio as $w_j$
  - Then we have:

\[
E(r_M) = \sum_{j=1}^{j=N} w_j (E(r_j) - r_f) + r_f
\]
\[
Var(r_M) = \sum_{j=1}^{j=N} \sum_{j=1}^{j=N} w_i w_j \text{Cov}(r_j, r_i)
\]
All Risk-Return Tradeoffs are Equal

- The excess return on an asset is:
  \[ (E(r_j) - r_f) \]
- Recall the relevant measure of asset risk relative to a well-diversified portfolio is the covariance. Thus, the risk of the asset is
  \[ \text{Cov}(r_j, r_M) \]
- Thus, the return per unit of risk is simply the ratio:
  \[ \frac{E(r_j) - r_f}{\text{Cov}(r_j, r_M)} \]
- This measure must be the same for all assets!
  » Consider what happens when this is not the case
  \[ \frac{E(r_A) - r_f}{\text{Cov}(r_A, r_M)} > \frac{E(r_B) - r_f}{\text{Cov}(r_B, r_M)} \]

The Capital Asset Pricing Model

- If the risk-return tradeoff is the same for all assets, then it is the one of the market:
  \[ \frac{E(r_A) - r_f}{\text{Cov}(r_A, r_M)} = \frac{E(r_B) - r_f}{\text{Cov}(r_B, r_M)} = \frac{E(r_M) - r_f}{\text{Var}(r_M)} \]

This gives the relationship between risk and expected return for individual stocks and portfolios.

» This is called the Security Market Line.

\[ E(r_A) = r_f + \frac{\text{Cov}(r_A, r_M)}{\text{Var}(r_M)} (E(r_M) - r_f) = r_f + \beta_A (E(r_M) - r_f) \]

where \[ \beta_A = \frac{\text{Cov}(r_A, r_M)}{\text{Var}(r_M)} \]
Capital Asset Pricing Model
A Graphical Illustration

Expected Return

Expected Market Return

Risk free rate

0 0.5 1.0 Beta

Expected return = Risk free rate + Beta factor x Expected market risk premium

The CML and SML

\[ E(r_p) = r_f + \beta_p \cdot \frac{E(r_M) - r_f}{\sigma_M} \]

\[ E(r_a) = r_f + \beta_a \left( E(r_M) - r_f \right) \]
The SML and mispriced stocks

- Suppose for a particular stock:
  \[ E(r_j) < r_f + \frac{\text{Cov}(r_j, r_M)}{\text{Var}(r_M)} [E(r_M) - r_f] \]

- Remember the definition of expected returns:
  \[ E(r_j) = \frac{E(P_j^1 + D_j^1) - P^0_j}{P^0_j} \]

- Then \( P_0 \) falls, so that \( E(r_j) \) increases until disequilibrium vanishes and the equation holds!

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The SML and mispriced stocks

- Stock \( j \) is overvalued at \( X \):
  - price drops,
  - expected return rises.

- At \( Y \), stock \( j \) would be undervalued:
  - expected return falls
  - price increases
The Capital Asset Pricing Model

- The appropriate measure of risk for an individual stock is its **beta**.
- Beta measures the stock’s sensitivity to **market risk** factors.
  - The higher the beta, the more sensitive the stock is to market movements.
- The average stock has a beta of 1.0.
- Portfolio betas are weighted averages of the betas for the individual stocks in the portfolio.

**Beta and Standard Deviation**

\[
\text{Risk of a Share (Variance)} = \text{Market risk of the share} + \text{Specific risk of the share} = \beta \text{ of share} \times \text{Risk of market} + \text{This is the major element of a share's risk}
\]

\[
\text{Risk of a portfolio} = \text{Market risk of the portfolio} + \text{Specific risk of the portfolio} = \beta \text{ of Portfolio} \times \text{Risk of market} + \text{This is negligible for a diversified portfolio}
\]
Using Regression Analysis to Measure Betas

Betas of Selected Common Stocks
(based on monthly returns)

<table>
<thead>
<tr>
<th>Stock</th>
<th>Beta (5 years)</th>
<th>Beta (10 years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GENERAL MOTORS CORP</td>
<td>1.18</td>
<td>1.10</td>
</tr>
<tr>
<td>BOEING CO</td>
<td>0.98</td>
<td>0.70</td>
</tr>
<tr>
<td>BRISTOL MYERS SQUIBB CO</td>
<td>0.59</td>
<td>0.56</td>
</tr>
<tr>
<td>DELTA AIR LINES INC</td>
<td>2.54</td>
<td>1.51</td>
</tr>
<tr>
<td>CONTINENTAL AIRLINES INC</td>
<td>3.60</td>
<td>2.15</td>
</tr>
<tr>
<td>HEWLETT PACKARD CO</td>
<td>1.72</td>
<td>1.70</td>
</tr>
<tr>
<td>DOW CHEMICAL CO</td>
<td>0.94</td>
<td>0.77</td>
</tr>
<tr>
<td>EXXON CORP</td>
<td>0.57</td>
<td>0.44</td>
</tr>
<tr>
<td>MERCK &amp; CO INC</td>
<td>0.40</td>
<td>0.37</td>
</tr>
<tr>
<td>A &amp; T &amp; T CORP</td>
<td>0.84</td>
<td>0.95</td>
</tr>
<tr>
<td>HOME DEPOT INC</td>
<td>1.38</td>
<td>1.18</td>
</tr>
<tr>
<td>MCDONALDS CORP</td>
<td>1.01</td>
<td>0.73</td>
</tr>
<tr>
<td>MICROSOFT CORP</td>
<td>1.18</td>
<td>1.45</td>
</tr>
<tr>
<td>APPLE COMPUTER INC</td>
<td>1.50</td>
<td>1.40</td>
</tr>
<tr>
<td>GOOGLE INC</td>
<td>-0.07</td>
<td>-0.07</td>
</tr>
<tr>
<td>WAL MART STORES INC</td>
<td>0.58</td>
<td>0.67</td>
</tr>
<tr>
<td>JPMORGAN CHASE &amp; CO</td>
<td>1.63</td>
<td>1.52</td>
</tr>
<tr>
<td>YAHOO INC</td>
<td>2.80</td>
<td>3.22</td>
</tr>
</tbody>
</table>
Beta and Volatility
(based on monthly returns)

<table>
<thead>
<tr>
<th>Company</th>
<th>Beta</th>
<th>Total StDev</th>
<th>Specific Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>GENERAL MOTORS CORP</td>
<td>1.18</td>
<td>10.01</td>
<td>8.51</td>
</tr>
<tr>
<td>BOEING CO</td>
<td>0.98</td>
<td>9.03</td>
<td>7.95</td>
</tr>
<tr>
<td>BRISTOL MYERS SQUIBB CO</td>
<td>0.59</td>
<td>7.87</td>
<td>7.41</td>
</tr>
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<td>DELTA AIR LINES INC</td>
<td>2.54</td>
<td>21.06</td>
<td>17.6</td>
</tr>
<tr>
<td>CONTINENTAL AIRLINES INC</td>
<td>3.6</td>
<td>22.83</td>
<td>16.2</td>
</tr>
<tr>
<td>HEWLETT PACKARD CO</td>
<td>1.72</td>
<td>11.63</td>
<td>8.72</td>
</tr>
<tr>
<td>DOW CHEMICAL CO</td>
<td>0.94</td>
<td>7.81</td>
<td>6.59</td>
</tr>
<tr>
<td>EXXON CORP</td>
<td>0.57</td>
<td>5.45</td>
<td>4.83</td>
</tr>
<tr>
<td>MERCK &amp; CO INC</td>
<td>0.4</td>
<td>8.05</td>
<td>7.85</td>
</tr>
<tr>
<td>A T &amp; T CORP</td>
<td>0.84</td>
<td>10.34</td>
<td>9.62</td>
</tr>
<tr>
<td>HOME DEPOT INC</td>
<td>1.38</td>
<td>8.69</td>
<td>6.12</td>
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<tr>
<td>MCDONALDS CORP</td>
<td>1.01</td>
<td>7.92</td>
<td>6.52</td>
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<tr>
<td>MICROSOFT CORP</td>
<td>1.18</td>
<td>9.16</td>
<td>7.48</td>
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<tr>
<td>APPLE COMPUTER INC</td>
<td>1.5</td>
<td>13.69</td>
<td>11.95</td>
</tr>
<tr>
<td>GOOGLE INC</td>
<td>-0.07</td>
<td>14.47</td>
<td>14.47</td>
</tr>
<tr>
<td>WAL MART STORES INC</td>
<td>0.58</td>
<td>5.83</td>
<td>5.22</td>
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<tr>
<td>JPMORGAN CHASE &amp; CO</td>
<td>1.63</td>
<td>9.52</td>
<td>6.16</td>
</tr>
<tr>
<td>YAHOO INC</td>
<td>2.8</td>
<td>17.39</td>
<td>12.67</td>
</tr>
</tbody>
</table>

Market StDev over this 5 year period was 4.47%.

Estimating the Expected Rate of Return on Equity

- The SML gives us a way to estimate the expected (or required) rate of return on equity.
  \[ E(r_j) = r_f + \beta_j [E(r_M) - r_f] \]
- We need estimates of three things:
  - Riskfree interest rate, \( r_f \)
    - The riskfree rate can be estimated by the current yield on one-year Treasury bills.
  - Excess return of market, \( [E(r_M) - r_f] \).
    - The market excess return can be estimated by looking at the historical difference between the return on stocks and the return on Treasury bills.
  - Beta for the stock, \( \beta_j \).
    - The betas are estimated by regression analysis. They are also published in beta books (e.g., Value Line, Merrill Lynch).
Estimating the Expected Rate of Return on Equity

\[ E(r) = 5.2\% + \beta (8.6\%) \]

<table>
<thead>
<tr>
<th>Company</th>
<th>Beta</th>
<th>E(r)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GENERAL MOTORS CORP</td>
<td>1.18</td>
<td>15.35</td>
</tr>
<tr>
<td>BOEING CO</td>
<td>0.98</td>
<td>13.63</td>
</tr>
<tr>
<td>BRISTOL MYERS SQUIBB CO</td>
<td>0.59</td>
<td>10.27</td>
</tr>
<tr>
<td>DELTA AIR LINES INC</td>
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<tr>
<td>CONTINENTAL AIRLINES INC</td>
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<tr>
<td>HEWLETT PACKARD CO</td>
<td>1.72</td>
<td>19.99</td>
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<td>DOW CHEMICAL CO</td>
<td>0.94</td>
<td>13.28</td>
</tr>
<tr>
<td>EXXON CORP</td>
<td>0.57</td>
<td>10.10</td>
</tr>
<tr>
<td>MERCK &amp; CO INC</td>
<td>0.4</td>
<td>8.64</td>
</tr>
<tr>
<td>A T &amp; T CORP</td>
<td>0.84</td>
<td>12.42</td>
</tr>
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<td>HOME DEPOT INC</td>
<td>1.38</td>
<td>17.07</td>
</tr>
<tr>
<td>MCDONALDS CORP</td>
<td>1.01</td>
<td>13.89</td>
</tr>
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<td>MICROSOFT CORP</td>
<td>1.18</td>
<td>15.35</td>
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<tr>
<td>APPLE COMPUTER INC</td>
<td>1.5</td>
<td>18.10</td>
</tr>
<tr>
<td>GOOGLE INC</td>
<td>-0.07</td>
<td>4.60</td>
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<tr>
<td>WAL MART STORES INC</td>
<td>0.58</td>
<td>10.19</td>
</tr>
<tr>
<td>JPMORGAN CHASE &amp; CO</td>
<td>1.63</td>
<td>19.22</td>
</tr>
<tr>
<td>YAHOO INC</td>
<td>2.8</td>
<td>29.28</td>
</tr>
</tbody>
</table>

Example of Portfolio Betas and Expected Returns

- Maintaining the assumption that the risk free rate is 5.2\% and the risk premium is 8.6\% : what is the beta and expected rate of return of an equally-weighted portfolio consisting of Exxon (beta = 0.57) and AT&T (beta = 0.84)?

- How would you construct a portfolio with the same beta and expected return, but with the lowest possible standard deviation?

- Use the figure on the following page to locate the minimum variance portfolio with the same expected return.
Example: Mispriced Portfolio

- You have information that the equal weighted portfolio of Exon and AT&T has an expected return that is lower than 11.3%.
  » How should you take advantage of this?
  » Is this an arbitrage opportunity?
  » What risk can you hedge here?
Example: Mispriced Portfolio

<table>
<thead>
<tr>
<th>Buy cheap:</th>
<th>( w_m = 0.71 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (1 - w_m) = 0.29 )</td>
<td>( \frac{1}{1} )</td>
</tr>
</tbody>
</table>

\[ \beta = 0.71 \]

<table>
<thead>
<tr>
<th>Sell expensive:</th>
<th>( w_{Exxon} = -0.5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_{AT&amp;T} = -0.5 )</td>
<td>( \frac{-1}{-1} )</td>
</tr>
</tbody>
</table>

\[ \beta = -0.71 \]

- Overall Investment: \( 1 - 1 = 0 \)
- Overall \( \beta \): \( 0.71 - 0.71 = 0 \)

Example

- The S&P500 Index has a standard deviation of about 12% per year.
- Gold mining stocks have a standard deviation of about 24% per year and a correlation with the S&P500 of about \( \rho = 0.15 \).
- If the yield on U.S. Treasury bills is 5.2% and the market risk premium is \( [E(r_m) - r_f] = 8.6\% \), what is the expected rate of return on gold mining stocks?
Example

- The beta for gold mining stocks is calculated as follows:

- The expected rate of return on gold mining stocks is:

- What portfolio has the same expected return as gold mining stocks, but the lowest possible standard deviation?

Graphical Illustration
Using the CAPM for Project Evaluation

- Suppose Microsoft is considering an expansion of its current operations.
  - The expansion will cost $140 million today
  - Expected to generate a net cash flow of $25 million per year for the next 20 years.
  - What is the appropriate risk-adjusted discount rate for the expansion project?
  - What is the NPV of Microsoft’s investment project?
  - Disregard the effect of leverage

Microsoft’s Expansion Project

- The risk-adjusted discount rate for the project, \( r_p \), can be estimated by using Microsoft’s beta and the CAPM.

\[
 r_p = r_f + \beta (E[r_m] - r_f) = 0.052 + 1.18(0.086) = 0.1535
\]

- Thus, the NPV of the project is:

\[
 NPV = \frac{25}{0.1535} \left(1 - \frac{1}{1.1535^{20}}\right) - 140 = 13.502 M
\]
Company Risk Versus Project Risk

- The company-wide discount rate is the appropriate discount rate for evaluating investment projects that have the same risk as the firm as a whole.

- For investment projects that have different risk from the firm’s existing assets, the company-wide discount rate is not the appropriate discount rate.

- In these cases, we must rely on industry betas for estimates of project risk.

Company Risk versus Project Risk

- Suppose Microsoft is considering investing in the development of a new airline.
  » What is the risk of this investment?

  » What is the appropriate risk-adjusted discount rate for evaluating the project?

  » Suppose the project offers a 17% rate of return. Is the investment a good one for Microsoft?
Industry Asset Betas

<table>
<thead>
<tr>
<th>Industry</th>
<th>Beta</th>
<th>Industry</th>
<th>Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Airlines</td>
<td>1.80</td>
<td>Agriculture</td>
<td>1.00</td>
</tr>
<tr>
<td>Electronics</td>
<td>1.60</td>
<td>Food</td>
<td>1.00</td>
</tr>
<tr>
<td>Consumer Durables</td>
<td>1.45</td>
<td>Liquor</td>
<td>0.90</td>
</tr>
<tr>
<td>Producer Goods</td>
<td>1.30</td>
<td>Banks</td>
<td>0.85</td>
</tr>
<tr>
<td>Chemicals</td>
<td>1.25</td>
<td>International Oils</td>
<td>0.85</td>
</tr>
<tr>
<td>Shipping</td>
<td>1.20</td>
<td>Tobacco</td>
<td>0.80</td>
</tr>
<tr>
<td>Steel</td>
<td>1.05</td>
<td>Telephone Utilities</td>
<td>0.75</td>
</tr>
<tr>
<td>Containers</td>
<td>1.05</td>
<td>Energy Utilities</td>
<td>0.60</td>
</tr>
<tr>
<td>Nonferrous Metals</td>
<td>1.00</td>
<td>Gold</td>
<td>0.35</td>
</tr>
</tbody>
</table>


Company Risk versus Project Risk

- The project risk is closer to the risk of other airlines than it is to the risk of Microsoft’s software business.
- The appropriate risk-adjusted discount rate for the project depends upon the risk of the project. If the average asset beta for airlines is 1.8, then the project’s cost of capital is:

\[
r_p = r_f + \beta_p (E[r_m] - r_f)
\]

\[
r_p =
\]
Company Risk versus Project Risk

Project Evaluation: Rules

- The risk of an investment project is given by the project’s beta.
  » Can be different from company’s beta
  » Can often use industry as approximation

- The Security Market Line provides an estimate of an appropriate discount rate for the project based upon the project’s beta.
  » Same company may use different discount rates for different projects

- This discount rate is used when computing the project’s net present value.
Summary

- Optimal investments depend on trading off risk and return
  » Investors with higher risk tolerance invest more in risky assets
  » Only risk that can’t be diversified counts
- If investors can borrow and lend, then everybody holds a combination of two portfolios
  » The market portfolio of all risky assets
  » The riskless asset
  - Covariance with the market portfolio counts
- In equilibrium, all stocks must lie on the security market line
  » Beta measures the amount of non-diversifiable risk
  » Expected returns reflect only market risk
  » Use these as required returns in project evaluation